Mathematical modelling of the combined optimization of a pumped-storage hydro-plant and a wind park

L. Bayón, J.M. Grau, M.M. Ruiz, P.M. Suárez

bayon@uniovi.es; grau@uniovi.es; mruiz@uniovi.es; pedrosr@uniovi.es

Department of Mathematics, E.P.I. Gijón, University of Oviedo, Spain

Spanish energy regulations have managed to position Spain as the second country worldwide with more wind power capacity. The new regulations (RD661/2007) allows wind farms to go to the market to sell the energy generated by their facilities. If the wind farms offer in the pool, they will behave like any other generating resource. Consequently they will prepare their offers, programming their power production and selling it in the pool.

However, wind power has a number of characteristics that make it special, the most important of which is the problem of the unpredictability of wind farm production. The main difficulty of predicting wind power is that the power is proportional to the cube of the wind speed. This means that small deviations in the speed generate large deviations in the power prediction. In the case of over- or undersupply, other producers must reduce or increase their production to resolve the so-called deviation, thereby incurring financial losses. These financial losses lead to what are known as deviation penalties, provided for in RD661/2007.

Faced with this situation, wind farms have several options: they can try to offer on the intraday spot markets to re-buy or sell energy and thus compensate for prediction errors; they can pay the penalties associated with non-compliance as regards energy supply; or they can try to store wind power energy in some way. Diverse methods have also been proposed to store this energy: compressed air storage, batteries for electric cars, flywheels, latent heat and sensible heat storage systems, superconducting magnets, hydrogen and, what we analyse in this paper, pumped water.

In this paper we consider the combined optimization of a pumped-storage hydro-plant and a wind park. First, we are going to present the mathematical modelling of the resulting problem. Second, we shall solve the optimization problem using Optimal Control techniques and finally we shall present an example.

It is very important to highlight the fact that the mathematical modelling is complicated, as there are many variables which influence the final result, like:

$[0,T]$: optimization interval (in h).

$z(t)$: volume of water discharge up to instant t (in m^3).

$z'(t)$: rate of water discharge at instant t (in m^3/s).

b : volume of water that must be discharged throughout the optimization interval $[0,T]$.

$P(t)$: hydraulic power production of the pumped-storage hydro-plant (in MW).

A : efficiency of the hydro-plant in generation zone (in $MW.s/m^3$)

η : efficiency of the pumping process.

$q_{\min}, q_{\max}$: technical constraints for $z'(t)$ (in m^3/s).

$\pi(t)$: clearing price at each hour t (in $euros/h.MW$).

$W(t)$: wind power production at instant t (in MW).

$C(t)$: cost of deviation penalties at instant t (in $euros/h$).

$D(t)$: deviations in the wind power prediction at instant t (in MW).

d: percentage of the clearing price applied to cost from deviation (in %).