Low rank acceleration of symmetric/nonsymmetric preconditioners for the nonlinear Richards equation

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We study Broyden-type low rank updates of an initial preconditioner for solving the sequence of linear systems arising from Newton-like linearizations of (Mixed or Galerkin) Finite Element-discretized Richards equation that models fluid flow in variably saturated porous media. Starting from the incomplete LU decomposition of the initial Jacobian matrix, we apply this approach to build a sequence of preconditioners. We also study the behavior of a symmetric positive definite (SPD) sequence of preconditioner obtained starting from an SPD approximation of the Jacobian as in the Picard Newton-like scheme. This will allow to use the Preconditioned Conjugate Gradient method for the solution of the linearized system.

Numerical experiments on realistic problems of large size show a reduction in the number of iterations needed to achieve convergence in the linear solver and in the cost of computing the preconditioner.