APPLICATION OF GRAPH-SPECTRAL METHODS IN THE VULNERABILITY ASSESSMENT OF WATER SUPPLY NETWORKS

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Abstract

Many critical infrastructures consist of networks with complex structure, since they have highly interdependent elements and nontrivial designs with a degree of complexity that depends on the settings and level of interaction between these elements. The theory of complex networks employs techniques borrowed from graph theory to classify different types of networks, analyse their structures, and quantify vulnerability and robustness against node/link failures and malicious attacks.

In the context of vulnerability assessment of complex networks, graph theory techniques facilitate the representation and analysis. A network is represented as a mathematical graph $G=G(V, E)$ in which V is the set of all graph nodes, with card(V) = n, and E is the set of the graph edges, with card(E) = m. Graphs are a useful abstraction to simulate real scenarios; to this purpose, nodes and edges must be suitably defined (i.e. pumping stations, tanks, pipes, etc.) depending on the intended type of network analysis. In general, complex networks can be classified as technological, biological, social or information networks. In addition, their topological structures can be categorized as centralized, decentralized and distributed depending on the hierarchical importance or redundant configurations (Newman, 2003). Also, graph theory is based on a set of measures to evaluate the networks. These measures can be classified as basic connectivity measures, spectral measures and statistical measures (Grubesic *et al.,* 2008).

In this paper we introduce a spectral methodology based on spectral measures to establish vulnerability areas in water supply networks (WSNs), which are highly distributed and a mixture of technological, biological and information networks. Spectral measures relate the network topology with the intensity of connectivity and cohesion graph, by analysing the spectrum of the adjacency matrix of the network. Among these measures, the *PageRank* algorithm measures the relative importance defined in any given graph. Herrera *et al.* (2011), has recently applied this methodology to WSNs management.

To achieve an efficient vulnerability analysis we propose a prospective study to create importance areas. To meet this objective, these areas are approached by a flexible method of semi-supervised clustering (Herrera *et. al.,* 2010). As a result, we will obtain a telescopic screening of the whole system vulnerabilities. This allows us achieve better control by working in a top-down way from the whole network to the detail of a particular sector, which will be specifically taken into account. This provides valuable additional support to the vulnerability assessment in WSNs management.

Keywords: Graph theory, vulnerability, spectral methods, PageRank, Water Supply Networks

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