Generating optimal iterative methods for non linear equations by using polynomial interpolation

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Abstract

In this work we show a general procedure to obtain optimal iterative methods [1] for nonlinear equations $f(x) = 0$, applying polynomial interpolation to a generic optimal iterative method of lower order.

Let us consider an optimal method of order $p = 2^n$, $y_k = \phi_p(x_k)$, that uses $n+1$ functional evaluations. Then, we perform a Newton-like step $z_k = y_k - \frac{f(y_k)}{f'(y_k)}$ which gives us a method of order 2^{n+1}, that is not optimal because it introduces two new functional evaluations. Instead, we approximate the derivative by using a polynomial of degree $n+1$ that interpolates $n+2$ already known functional values and keeps the order 2^{n+1}.

We have applied this idea to Newton and Steffensen's methods, [2], obtaining optimal methods of order 4, 8 and 16.

In addition, we provide different numerical tests, which confirm the theoretical results.

References

[1] H.T. Kung, J.F. Traub, *Optimal order of one-point and multi-point iteration*, Applied Mathematics and Computation, 21 (1974) 643-651.

[2] J.M. Ortega, W.G. Rheinboldt, Iterative solutions of nonlinear equations in several variables, Academic Press, New York, 1970.

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