On the Application of Singular Value Decomposition and Tikhonov Regularization to Some Ill-Posed Problems in Hyperbolic Passive Location

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***Abstract*—Hyperbolic passive location of targets (e.g. used in the Multilateration systems for civil aviation) calls for the solution of highly non-linear systems. In this paper we show how the effect of typical ill-conditioning of passive location, arising in many operational conditions, may be significantly mitigated by regularization techniques. The pertaining advantage regarding to the conventional (non-regularized) approach are shown in a number of realistic situations.**

The hyperbolic location systems are a powerful surveillance tool for air traffic control and airport management (taxiing, taking off / landing, approach or enroute) and in this field they are based on the reception, by a number of stations, of signals emitted by a standard on-board transponder, the Secondary Surveillance Radar (SSR) Selective Mode or Mode-S. These systems are intended to inform air traffic controllers of the location and identification of aircraft or vehicles equipped with an operational SSR transponder [1], and perform the localization calculations by solving a system of hyperbolic equations based on TDOA (Time Difference of Arrival) technique; the algorithms for solving them run at real time in a central computer.

The unknown target position (i.e. of the transponder antenna) is non-linearly related to the measurements; the location of the target calls for iterative solution techniques where the equations are linearized and the first iteration is based on a suitable "guess". In some scenarios, especially when the target is outside the perimeter of the various stations, it is common to find a typical problem; i.e., the system of equations is ill conditioned [2]. This problem is known in the literature as an ill-posed problem. The consequence of this is that, when the system of equation is solved, the solution (target position) has a big error that may diverge [3]. On the other hand, the effects of this problem in the multilateration systems accuracy have been highlighted in [4-5].

Nowadays, the most established strategy to solve the system of hyperbolic equations, is based on an iterative procedure that uses a Taylor linearization [6-7] with the pseudoinverse matrix to solve the resulting linear system. However, as it can be shown in the accuracy analysis presented in [5], this strategy does not provide acceptable accuracies when the problem is ill-posed.

In this paper we study, analyze and develop two methods, one, based on Singular Value Decomposition (SVD), as the well known T-SVD (Truncated SVD) [8] and the other one based on the Tikhonov regularization [9], to solve that iterative procedure commented above. The influence of the error due to the measurements noise, to the linearization of the problem and to the system geometry, in the ill-posed problem, is analyzed by means of the SVD spectrum of the coefficient matrix. Moreover, some guidelines to choose the regularization parameter value (the discrete one for T-SVD and the continuous one for Tikhonov) are commented. Finally, some simulations results are shown and discussed.

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