Modeling a stock loan pricing with an obstacle problem associated to a Kolmogorov equation

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Abstract

A stock loan is a contract between a lender and a borrower. The borrower owns a share of a stock which acts as the collateral of the loan obtained from the lender. At any time before or at loan maturity, the borrower may recover the stock by repaying the lender the principal and the fixed interest rate associated to the loan. Otherwise, the borrower can surrender the stock instead of paying the loan. There are different contract specifications concerning the destination of the dividends associated to the stock: they can be either gained by the lender or by the borrower, also either before or on redemption, as indicated in [2] and [4].

In this work we present a mathematical model to price a stock loan when the accumulative dividend yield is returned to the borrower on redemption. In this case, the introduction of a path dependent variable allows to pose an obstacle problem associated to an ultraparabolic PDE of Kolmogorov type, as in the Asian options with continuous arithmetic averaging. Existence and uniqueness can be analyzed with the techniques developed in [3]. As the analytical solution cannot be obtained, we propose a numerical method to approximate the solution based on the techniques introduced in [1]. More precisely, a Crank-Nicolson Lagrange-Galekin method combined with an Augmented Lagrangian Active Set technique to deal with the free boundary problem are used. Finally, some examples to illustrate the performance of the numerical methods are presented.

References

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