New advances on matrix exponential computation for engineering problems

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ABSTRACT

Many engineering processes are described by systems of linear first-order ordinary differential equations with constant coefficients, whose solutions are given in terms of the matrix exponential, and, in many times, they involve large matrices. Many methods for matrix exponential computation have been proposed [1, 2]. From all the methods, the squaring and scaling technique is one of the most widely used. Unfortunately, the matrix size is not taken into account in the scaling and squaring state-of-the-art methods [4, 5, 3].

This work improves the scaling and squaring algorithm presented in [3] providing a competitive scaling and squaring algorithm for matrix exponential computation. The new algorithm employs an improved version of Theorem 1 from [3] to bound the norm of matrix power series. We show that under the same conditions the new theorem provides lower values of the scaling parameter s than Theorem 4.2 from [5], reducing the cost. We provide a new formula for the forward relative error of the algorithm in exact arithmetic. Using that formula and new error bounds for normal and nonnormal matrices we give new sharp bounds for forward and backward relative errors. Moreover, the matrix size is used to improve the scaling parameter selection reducing even more the cost and improving the accuracy. We provide a Matlab version of the new algorithm, and a comparison with the original algorithm from [3] and the state-of-the-art algorithm from [5], obtaining excellent accuracy and processing time results.

References

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