Preconditioning the solution of the time dependent neutron diffusion equation by recycling Krylov subspaces.

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After the spatial discretization of the neutron diffusion equation, a semidiscrete system of ordinary differential equations is obtained. This is a stiff system of differential equations, where the matrices involved are large and sparse. Usually, this system is solved using an implicit time discretization, which implies to solve an algebraic system of linear equations for this time step. This task is carried out by means of iterative Krylov subspace methods. The converge rate of these methods can largely be improved if a suitable preconditioner is used [2]. Usual preconditioners are based on incomplete factorizations of the system matrix, but this approach is expensive in terms of the computational time and memory for the storage of the matrix.

The convergence of the Krylov methods generally depends on the eigenvalues and eigenvectors of the coefficients matrices. When some estimations of eigenvectors and eigenvalues are available, low rank transformations can be applied to improve the convergence rate the iterative method. This technique is known as spectral preconditioning of linear systems [3].

For a given transient, a number of systems have to be solved whose matrices varies continuously in time, and for successive time steps the coefficients matrices are expected to have similar eigenvalues and eigenvectors. Thus, the information obtained from the Krylov subspace when solving a system, can be used to precondition the system corresponding to the next time step. Different spectral preconditioners based on different Krylov methods such as GMRES-DR [1] and GCRO-DR [4] will be proposed and studied using a typical transient in a nuclear power reactor with hexagonal geometry.

References

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