Wavelet based approach for singular perturbation problems

D. Černá, V. Finěk

E-mail: dana.cerna@tul.cz, vaclav.finek@tul.cz

Department of Mathematics and Didactics of Mathematics,
Technical University of Liberec,
Studentská 2, 461 17 Liberec, Czech Republic

We consider here the following singularly perturbed two-point boundary value problem

$$-\epsilon u''(x) + p(x)u'(x) + q(x)u(x) = f(x) \quad \text{for } x \in (a, b) \text{ and with } u(a) = \alpha,\ u(b) = \beta,$$

where a, b, α, β are constants and $0 < \epsilon \ll 1$. This equation represents simple mathematical model of a convection-diffusion problem and it can be used to model many practical problems. For example linearized Navier-Stokes equations at high Reynolds number provides an accurate model of dynamics of transition in the problem of turbulence suppression in channel flow. Problems of this types have solutions which are discontinuous as ϵ is approaching to zero and typically possess boundary or interior layers, i.e. regions of rapid change in the solution near the endpoints or some interior points. Many numerical methods have been suggested to solve such types of problems and a lot of them requires informations about locations and widths of different layers. One of possibilities, how to solve them without these informations, is an application of adaptive methods. We employ here an asymptotically optimal adaptive wavelet scheme. It generates the approximate solution comparable with the best N-term approximation and the number of arithmetic operations needed to compute this solution is proportional to N. Next advantage of this scheme consists in the efficient diagonal preconditioning. The condition number of arising stiffness matrices is bounded independently of matrix size. To further improve the quantitative properties of the used scheme and simplify implementation, we construct a multiwavelet basis on the interval based on Hermite cubic splines with respect to which both the mass and stiffness matrices corresponding to the one-dimensional Poisson equation are sparse. This basis can be easily extended to n dimensions by tensor product. Moreover, constructed wavelets have high order of vanishing moments. Finally, we show some numerical experiments to demonstrate this approach.