A comparative analysis between some iterative methods from a dynamical point of view *

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Many applied problems in different fields of science and technology require to find the solutions of a nonlinear equation $f(z) = 0$, with $f : D \subseteq \mathbb{C} \to \mathbb{C}$. In the last years there exists a real competition between researchers to construct improved iterative methods for solving nonlinear equations. The application of these iterative schemes for solving $f(z) = 0$ gives rise to rational functions whose dynamics are not well-known. The simplest model is Newton's method whose fixed point function is $N_f(z) = z - \frac{f(z)}{f'(z)}$. If $p(z)$ is a complex polynomial, then the function $N_p(z)$ defines a rational map on the Riemann sphere $\bar{\mathbb{C}}$, and hence defines a discrete dynamical system $z_{n+1} = N_p(z_n)$.

The dynamics of Newton's scheme has been studied widely (see, for instance, [1]). This study has been extended to other point-to-point iterative methods, with convergence order up to three (see, for example, [2]).

From the numerical point of view, the dynamical behavior of the rational function associated to an iterative method give us important information about its stability and reliability. Given a rational function we are interested in the study of its fixed and critical points, the asymptotic behavior of the orbits depending on the initial condition z_0, the basins of attraction of the different roots and the description of the Julia and Fatou sets associated to the rational function.

In this paper, we analyze the mentioned complex dynamics elements for some iterative Newton-type and Steffensen-type methods applied to several complex polynomials in order to compare them in terms of stability, wideness of the convergence regions and possible regions of attraction different than the basins of attraction (the infinity as a superattractor, the existence of attractive periodic orbits,...).

References

[1] P. Blanchard, The Dynamics of Newton's method, Proc. of Symposia in Applied Math., 49 (1994) 138-154.

[2] J.M. Gutiérrez, M.A. Hernández and N. Romero, Dynamics of a new family of iterative processes for quadratic polynomials, Journal of Computational and Applied Mathematics, 233 (2010) 2688-2695.

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