The effect of the memory on the spread of a disease through the environtment

C. Coll, D. Ginestar, A. Herrero [1] and E. Sánchez

Institut Universitari de Matemàtica Multidisciplinar,
Universitat Politècnica de València .

Abstract

In recent years, several papers have studied how to apply the fractional order derivative to model the spread of an infectious disease. Most of them apply Caputo's definition of fractional derivative in differential equations or systems of differential equations [1]. However, it is interesting to work with discrete systems since statistical data on epidemics are collected at discrete times and it is easier to compare them with the output of discrete-time systems. For the discrete case, the fractional order differential operator is less used. In this line, some papers study discrete linear systems of fractional order using the discrete approximation of the Grünwald-Letnikov fractional derivative, see for instance [2].

In this paper, we propose a mathematical representation to study the behaviour of the solution of an epidemic model in which the disease is transmitted through the environtment. We focus on an epidemic process that includes indirect transmission of the disease when the population comes into contact with the underlying contamination in space. Then, in addition to susceptible and infected individuals, we consider a new variable representing the amount of contaminant in the enclosure. We propose a discrete-time model based on fractional calculus where the state depends on the current state and previous ones. This model uses a discrete version of the Grünwald-Letnikov fractional derivative operator with truncated memory. This follows from the properties of the coefficients of our operator since we show that they can be negligible beyond a certain step. This justifies taking a truncated operator as a good approximation of the proposed process.

We obtain the equilibrium points of the proposed model and perform a thorough analysis of their stability. To determine whether the system is stable or not we define some parameters depending on the fractional order and the memory steps considered. In the analysis we observe that the population size plays an important role in ensuring the disappearance or permanence of the disease.

Finally, we illustrate different properties of the proposed model by analyzing different examples as a function of the fractional order and the number steps in which we truncate.

References

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[1] aherrero@mat.upv.es