Numerical computation of the 1-pdf for a differential equation with a single random parameter

ABSTRACT

The work developed in this research presents a numerical methodology for the treatment of random differential equations when it is not possible to obtain an analytical solution. The approach used allows the calculation of parameters such as the mean and variance of the solution function for different time instants. One of the advantages of the methodology is its low computational requirement and the stability of the results obtained for both parameters. Likewise, in the process, estimates for the probability density functions in the solution space are obtained in a simple and direct way while determining the curves corresponding to different values of the desired percentiles.

The calculation is based on calculating the solution to the differential equation as a function that depends on both the independent variable and the random variable, considering together with the solutions of the original differential equation the estimated solution of a differential equation that is derived from the original one and that involves the derivative z(t) of such solution with respect to the random variable for different fixed values of the independent variable t. Thus, a set of solutions for x(t) and z(t) associated to a discretization of the product of domains of the independent variable and the random variable is generated. It is from these approximate solutions defined on the network of nodes that estimates can be obtained for the density functions of the variable X(t) and the corresponding mean and variance at each instant.