Maximum-Likelihood Expectation-Maximization method applied to unfold neutron spectra in a radiotherapy bunker

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Abstract

The Maximum-Likelihood Expectation-Maximization (MLEM) method is an iterative computation of maximum-likelihood estimation widely used to solved ill-posed problems [1]. The mathematical term ill-posed problem is used to denote problems that does not have a unique, stable, or well-defined solution. These conditions are usually due to the information provided to solve the problem is imprecise or incomplete which is common in some physics and engineering fields [2, 3].
An example of this, in the field of medical physics, is the neutron spectrum unfolding via the raw data measured with a Bonner Sphere Spectrometer (BSS) [4]. This device consists of a central thermal neutron detector and six high-density polyethylene spheres with different diameters. Each sphere moderator is inserted in the detector, which is positioned at the center of the sphere, being sensitive to different energy neutrons. The measurements obtained with the set of spheres are considered as the raw data, and each of them is denoted by m_i. Each sphere-detector combination i, has a unique response function depending on the neutron energy E, which is denoted by $R_i(E)$. The relation with neutron spectrum $n(E)$ to unfold, and the response function is given by the Fredholm integral equation of the first kind,

$$m_i = \int_E^{E+\Delta E} R_i(E)n(E)dE, \qquad \text{and written in a matrix form: } m = R \cdot n$$

Since in the described problem the number of measurements carried out, one for each sphere, is smaller than the number of energy bins of the unknown spectrum, 29 bins, the problem has an infinite number of mathematical solutions, some of them without a physical sense. By this reason, the MLEM method is suitable to obtain an accurate neutron spectrum. In this work, the neutron spectra at different locations of a external radiotherapy bunker have been obtained applying the MLEM with the aim to measure the neutron ambient dose produced in a radiotherapy treatment.

References

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