Parametric family of derivative-free multi-step vectorial methods with weight function

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Abstract

In this work, we present a parametric family of multi-step derivative-free iterative methods with a weight function for solving systems of nonlinear equations. This family, denoted by Mn, performs n steps on each iteration and has the following iterative expression:

$$\begin{cases} z_1^{(k)} = x^{(k)} - [w^{(k)}, x^{(k)}; F]^{-1} F(x^{(k)}), \\ z_2^{(k)} = z_1^{(k)} - H(t^{(k)})[w^{(k)}, x^{(k)}; F]^{-1} F(z_1^{(k)}) \\ \quad \vdots \qquad\qquad \vdots \\ z_j^{(k+1)} = z_{j-1}^{(k)} - H(t^{(k)})[w^{(k)}, x^{(k)}; F]^{-1} F(z_{j-1}^{(k)}), \quad j = 2, \ldots, n-1 \\ \quad \vdots \qquad\qquad \vdots \\ x^{(k+1)} = z_{n-1}^{(k)} - H(t^{(k)})[w^{(k)}, x^{(k)}; F]^{-1} F(z_{n-1}^{(k)}), \end{cases} \tag{1.1}$$

being $w^{(k)} = x^{(k)} + \beta F(x^{(k)})$, $v^{(k)} = z_1^{(k)} + \delta F(z_1^{(k)})$, $t^{(k)} = [w^{(k)}, x^{(k)}; F]^{-1}[z_1^{(k)}, v^{(k)}; F]$, H a real matrix weight function and $n \geq 3$.

It is proven that this family of iterative methods has order $2n$, when n steps are performed, as long as it is verified that $H(I) = I$ and $H'(I) = -I$. Moreover, if $H''(I) = 2I$, then that error $e_{k+1} = x^{(k+1)} - \alpha$ of M_n behaves as follows

$$e_{k+1} \sim (I - \delta F'(\alpha))(2I - \delta F'(\alpha))^{n-2}(I + \beta F'(\alpha))^n e_k^{2n},$$

where α is a solution of $F(x) = 0$ and $F'(\alpha)$ the Jacobian matrix of F evaluated at α. If $(2I - \delta F'(\alpha))^{n-2}(I + \beta F'(\alpha))^n = 0$ we could increase the order of the iterative method, but we do not know α, so we cannot define $\beta = -F'(\alpha)^{-1}$ or $\delta = 2F'(\alpha)^{-1}$, so we look for a good approximation of this Jacobian matrix. We introduce memory to the iterative class by approximating the Jacobian matrix using different divided difference operators. We carry out numerical experiments with the proposed methods to analyse their behaviour and compare them.

References

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