An Ulm-type iterative method for solving Volterra integral equations of the second kind

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Abstract

We consider a Volterra integral equation of the second kind [1],

$$u(x) = f(x) + \int_a^x K(x,t)u(t)\,dt, \quad x \in [a,b],$$

where $-\infty < a < x < b < +\infty$, $f(x) \in C[a,b]$ is a given function, $K(x,t)$ is the kernel of the integral equation and $u(x) \in C[a,b]$ is the unknown function to determine.

From the Lagrange interpolation, we transform the previous Volterra integral equation into a system of equations, that allows finding $u(\xi_i)$, where the nodes $\xi_i \in [a,b]$ are the zeros of Chebyshev, so that we can obtain a good approximation of the solution $u(x)$ by considering a sufficient number of nodes. Since, in general, a large number of nodes is needed, we consider the zeros of Chebyshev, that avoid the phenomenon of Runge in the interpolation process. Furthermore, if we consider a large number of nodes, the system obtained has a high order, that will be solved by an Ulm-type iterative method [2, 3].

References

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