High order geometric integrators for Hamiltonian problems

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Abstract

We present different families of new Runge-Kutta-Nyström (RKN) symplectic splitting methods of order 8, especially designed for second-order systems of ordinary differential equations of the form y'' = f(y). They show a better efficiency than state-of-the-art symmetric compositions of 2nd-order symmetric schemes and RKN splitting methods of orders 4 and 6 for medium to high accuracy. For some particular examples, they are even more efficient than extrapolation methods for high accuracies and integrations over relatively short time intervals, whilst preserving qualitative features of the system.

We analyze in detail the construction process: it is based on the use of the Lie formalism to obtain the order conditions, what are then numerically solved by using continuation methods implemented in Python.

This is a joint work with Sergio Blanes (Universitat Politècnica de València) and Fernando Casas (Universitat Jaume I)

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