A dynamical overview of the Chebyshev-Halley of rootfinding methods

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Abstract

In this work we analyze some aspects of the dynamics of the iteration functions that result by applying the iterative methods of the Chebyshev-Halley family ([1], [4]) to complex variable polynomials. The fundamental interest of applying this type of iterative processes is to approximate numerically the roots of the considered polynomial. However, as it was pointed out in [2] or [3] for instance, sometimes undesirable situations appear from the rootfinding point of view, such as strange fixed points (fixed points of the iteration function that are not roots of the polynomial) or cycles of length greater than one. The objective of this work is to characterize polynomials, known as "bad polynomials", for which there are methods of the Chebyshev-Halley family that give rise to one of these two behaviors. Moreover, we also prove the existence of "super bad polynomials", for which there is an iterative method whose iteration function has both strange fixed points and attractor cycles.

References

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