Modelling by using Graph Boolean Dynamical Systems

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Abstract

Graph Boolean Dynamical Systems, also known as Boolean Networks (BN), are network models where the elements are represented by nodes, whose state values are Boolean (i.e. in the set $\{0,1\}$), and the relations among them by arcs. The evolution update of these systems is formalized by Boolean functions, and can be synchronous (Parallel Dynamical Systems, PDS [1]) or asynchronous (Sequential Dynamical Systems, SDS [2]).

As the first examples of BN we find the cellular automata (CA) and the Kauffman networks (KN), which arose at the end of the sixties in the works [4] and [3], and whose development has been of a great relevance in the fields of the Theory of Computation and the Biomathematics, respectively. Later, they have allowed to model the behaviour of several phenomena in the fields of physics, chemistry, biology (particularly in areas such as genetics, ecology and epidemiology), sociology, artificial processes (such as computer simulation and encryption), etc. In recent years, other issues such as stability, controllability and observability have been profusely studied, obtaining some noteworthy results by using novel methods such as the semi-tensor product of matrices.

In this work we revise some important features of BN, specifically regarding their orbital structure. Note that knowing the periodic structure is crucial to understand the asymptotic behaviour of the system. We also present some examples to illustrate how these systems can model different problems.

References

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