Backward stability in rational eigenvalue problems solved via linearizations

Froilán Dopico[♭],[1] María C. Quintana[♮], and Paul Van Dooren[♢]

(♭) Universidad Carlos III de Madrid,
Avda. Universidad 30, 28911 Leganés, Spain.
(♮) Aalto University,
Otakaari 1, Espoo, Finland.
(♢) Université catholique de Louvain,
Avenue Georges Lemaitre 4, Louvain-la-Neuve, Belgium.

Abstract

Rational eigenvalue problems have received considerable attention in the last decade, since they appear in many applications and they are used to approximate general nonlinear eigenvalue problems. Each rational eigenvalue problem is defined via a rational matrix $R(\lambda)$, i.e., a matrix whose entries are rational functions in the variable λ, which can be represented in different forms. We study the backward stability of running a backward stable eigenstructure solver on a pencil $S(\lambda)$ that is a strong linearization of an arbitrary rational matrix $R(\lambda)$ expressed in the form $R(\lambda) = D(\lambda) + C(\lambda I_\ell - A)^{-1}B$, where $D(\lambda)$ is a polynomial matrix and $C(\lambda I_\ell - A)^{-1}B$ is a minimal state-space realization. We consider the family of block Kronecker linearizations of $R(\lambda)$, which have the following structure

$$S(\lambda) := \begin{bmatrix} M(\lambda) & \widehat{K}_2^T C & K_2^T(\lambda) \\ B\widehat{K}_1 & A - \lambda I_\ell & 0 \\ K_1(\lambda) & 0 & 0 \end{bmatrix},$$

where the blocks have some specific structures. Backward stable eigenstructure solvers applied to $S(\lambda)$ will compute the exact eigenstructure of a perturbed pencil $\widehat{S}(\lambda) := S(\lambda) + \Delta_S(\lambda)$ and the special structure of $S(\lambda)$ will be lost. In order to link this perturbed pencil with a nearby rational matrix, we construct a strictly equivalent pencil $\widetilde{S}(\lambda) = (I - X)\widehat{S}(\lambda)(I - Y)$ that restores the original structure, and hence is a block Kronecker linearization of a perturbed rational matrix $\widetilde{R}(\lambda) = \widetilde{D}(\lambda) + \widetilde{C}(\lambda I_\ell - \widetilde{A})^{-1}\widetilde{B}$, where $\widetilde{D}(\lambda)$ is a polynomial matrix with the same degree as $D(\lambda)$. Moreover, we bound appropriate norms of $\widetilde{D}(\lambda) - D(\lambda)$, $\widetilde{C} - C$, $\widetilde{A} - A$ and $\widetilde{B} - B$ in terms of an appropriate norm of $\Delta_S(\lambda)$. These bounds may be inadmissibly large, but we introduce a scaling that allows us to make them satisfactorily tiny. Thus, for this scaled representation, we prove that the QZ algorithm computes the exact eigenstructure of a rational matrix $\widetilde{R}(\lambda)$ that can be expressed in exactly the same form as $R(\lambda)$ with the parameters defining the representation very near to those of $R(\lambda)$. This shows that this approach is backward stable in a structured sense.

[1]dopico@math.uc3m.es