On doubly quasi-stochastic combined matrices

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Abstract

A matrix $U = (u_{ij}) \in \mathbb{R}^{n\times n}$ is said to be a doubly quasi-stochastic matrix if $\sum_{i=1}^{n} u_{ij} = 1$, for all $j = 1, 2, \ldots, n$ and $\sum_{j=1}^{n} u_{ij} = 1$, for all $i = 1, 2, \ldots, n$. The combined matrix of a nonsingular matrix A is the matrix $C(A) = A \circ A^{-T}$ where $\circ$ means the Hadamard (entrywise) product, and A^{-T} is the inverse transpose $(A^T)^{-1}$ of A.

Combined matrices have many applications in science, for instance in chemical and in control theory, they are called relative gain array (RGA), see [1, 2, 5]. It is easy to prove that the combined matrices are doubly quasi-stochastic matrices.

In [4] the authors studied the problem of characterizing when three real numbers u_{11}, u_{22} and u_{33}, are the diagonal entries of a 3×3 combined matrix U in various classes of matrices. In particular, they consider the class of totally positive matrices which all their minors are positive, (see [3, 6] and the references therein). Now, in this work we try to extend the characterization when nine real numbers u_{ij}, $1 \leq i, j \leq 3$ are given as the entries of a doubly quasi-stochastic matrix U, such that, $U = C(A)$ where A is a totally positive matrix.

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