Recent applications of the Minimally Implicit Runge-Kutta (MIRK) methods in astrophysics

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Abstract

We present the Minimally Implicit Runge-Kutta (MIRK) methods for the numerical evolution of two systems of hyperbolic equations with stiff source terms.

First, we show how the MIRK methods can be used for the numerical evolution of the resistive relativistic magnetohydrodynamic (RRMHD) equations, following the approach proposed by [1] of an augmented system of evolution equations to numerically deal with constraints. Previous approaches rely on Implicit-Explicit (IMEX) Runge-Kutta schemes; in general, compared to explicit schemes, IMEX methods need to apply an iterative process for the recovery (which can be very expensive computationally) of the primitive variables from the conserved ones in numerous additional times, which could also have potential convergence problems. The use of the MIRK methods minimize the number of recoveries needed. Two numerical tests are shown as illustrative examples.

Second, we show how the MIRK methods can be used for the numerical evolution of the evolution equations of radiation hydrodynamics in which reactions between matter and radiation couples the hydrodynamic equations to those of radiative transfer [2, 3]. The numerical treatment has to account for their potential stiffness, for example in optically thick environments. We show the results of applying the MIRK methods to the reactions between neutrinos and matter in core-collapse supernovae simulations.

In both systems of equations the MIRK methods are able to deal with stiff source terms producing stable numerical evolutions and taking into account the behavior of the evolved variables in the limit of the stiff regime. The inversion of the implicit operator in both cases can be done analytically and the resulting numerical method resembles a pure explicit scheme with an effective time-step. These methods can be potentially applied to other systems of hyperbolic equations with stiff source terms, when these stiff source terms allow a kind of linearization in terms of the evolved variables.

References

[1] Komissarov, S.S. Multidimensional numerical scheme for resistive relativistic magnetohydrodynamics *Monthly Notices of the Royal Astronomical Society*, 382(3):995–1004, 2007. DOI: 10.1111/j.1365-2966.2007.12448.x

[2] Munier, A., Weaver, R. Radiation transfer in the fluid frame: A covariant formulation: Part I: Radiation hydrodynamics *Computer Physics Reports*, 3:127–164, 1986. DOI: 10.1016/0167-7977(86)90007-9

[3] Munier, A., Weaver, R. Radiation transfer in the fluid frame: A covariant formulation: Part II: The radiation transfer equation *Computer Physics Reports*, 3:165–208, 1986. DOI: 10.1016/0167-7977(86)90008-0

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