An efficient method to compute the matrix exponential based on Chebyshev polynomials

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Abstract

Chebyshev polynomials of first kind $T_n(x)$ are defined explicitly as

$$T_n(x) \;=\; \sum_{k=0}^{\left[\frac{n}{2}\right]} \binom{n}{2k} \left(x^2 - I_r\right)^k x^{n-2k} \;,\; n \geq 0,$$

or using the following three-term-recurrence formula:

$$\left.\begin{array}{rcl} T_0(x) & = & 1 \\ T_1(x) & = & x \\ T_{n+1}(x) & = & 2xT_n(x) - T_{n-1}(x), n \geq 1 \end{array}\right\}$$

Some properties of this type of polynomials can be found in references [1, 2].

On the other hand, the well-known matrix exponential $e^A, A \in \mathbb{C}^{n\times n}$, appears in different problems of applied mathematics, physics, and engineering, see [3] and, for instance, in the solution of systems of linear constant coefficient ordinary differential equations. Several state-of-the-art algorithms have been provided for approximating e^A, see for example [4] and references therein.

In this work, we have developed the proposed method in [5] to compute the matrix exponential by using an approximation based on Chebyshev's polynomials. We have performed numerical experiments comparing implementations based on this algorithm with other ones of the state of the art [4]. It can be observed that, in general, these new implementations present higher accuracy than the other methods for the tests.

References

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