On the use of Euler polynomials to approximate the matrix cosine

José M. Alonso[♮1], Emilio Defez[*], Javier Ibáñez[*] and Jorge Sastre[♭]

(♮) Instituto de Instrumentación para Imagen Molecular,
(*) Instituto de Matemática Multidisciplinar,
(♭) Instituto Universitario de Telecomunicación y Aplicaciones Multimedia,
Universitat Politècnica de València. Camino de Vera s/n, 46022, Valencia, Spain.

Abstract

Euler's polynomials $E_n(x)$ are defined as the coefficients of the generating function

$$\frac{2e^{tx}}{e^t+1} = \sum_{n\geq 0} \frac{E_n(x)}{n!} t^n \ , \ |t| < \pi,$$

see formula 24.2.6 from [1, p. 588]. Using these polynomials, a method to approximate the exponential matrix e^A for $A \in \mathbb{C}^{r\times r}$ was developed in reference [2].

In this work, two expressions for the matrix cosine are addressed:

$$\cos(A/\lambda) = \frac{1}{2}\left(\cos(1/\lambda)+1\right)\sum_{n\geq 0}\frac{(-1)^n E_{2n}(A)}{\lambda^{2n}(2n)!} - \frac{1}{2}\sin(1/\lambda)\sum_{n\geq 0}\frac{(-1)^n E_{2n+1}(A)}{\lambda^{2n+1}(2n+1)!}, \quad (1.1)$$

$$\cos(A/\lambda) = \cos(1/2\lambda)\sum_{n\geq 0}\frac{(-1)^n E_{2n}\left(A+\frac{1}{2}I\right)}{\lambda^{2n}(2n)!}, \quad (1.2)$$

where λ is a real parameter. A novel numerical method based on formulas (1.1)-(1.2) to compute the matrix cosine has been designed and implemented. We have performed tests comparing its numerical performance with that of other state-of-the art algorithms [3]. It can be observed that, in general, these new implementations offer higher accuracy than the other methods for the numerical experiments.

References

[1] Olver F. W., Lozier D. W., Boisvert R. F., Clark C. W.(Eds.). NIST handbook of mathematical functions hardback and CD-ROM. Cambridge University Press, 2010.

[2] Alonso J. M., Ibáñez J., Defez E. and Alonso-Jordá P. Euler polynomials for the matrix exponential approximation. *Journal of Computational and Applied Mathematics*, 115074, 2023.

[3] Al-Mohy A. H., Higham N. J. and Relton S. D., New algorithms for computing the matrix sine and cosine separately or simultaneously. *SIAM Journal on Scientific Computing*, 37(1), A456-A487, 2015.

[1] jmalonso@dsic.upv.es