

# Parametric Iterative method for addressing an embedded steel constitutive model with multiple roots

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## Abstract

In this work we present an iterative procedure able to find the solution of a nonlinear structural model that presents different multiplicities where parametric coefficients are randomly selected inside a solvability region. To get this aim, we modify a multipoint fixed point class of iterative schemes, designed for single roots [1], to find multiple roots, which reaches fourth order of convergence. This family is derived from one previously designed for finding simple roots of nonlinear equations. As this class includes a great amount of elements with the same order of convergence, we use complex discrete dynamics techniques to select those members with more stable performance. Once this aim is achieved, the structural problem is satisfactorily solved, and also some academical problems in order to test its robustness and applicability.

Several authors [2, 3, 4] consider a steel reinforcement stiffened by the concrete bonded to it (or also called as “embedded bar model”). One of these approaches, the so-called Refined Compression Field Theory (RCFT) [5], includes in the steel constitutive model an equilibrium condition that takes into account the concrete tension stiffening effect between cracks. As result, a nonlinear equation is introduced in the steel constitutive model in terms of the apparent yield strain. This last theory predicts the average stress of an embedded bar as a function of the average strain (i.e., measured on certain length including several cracks) such as follows:

$$\sigma_{s,av} = \begin{cases} f_y - \frac{A_c}{A_s} \frac{f_{ct}}{1 + \sqrt{3.6M\varepsilon_{s,av}}} & \text{if } \varepsilon_{s,av} \geq \varepsilon_{max}, \\ E_s \varepsilon_{s,av} & \text{if } \varepsilon_{s,av} < \varepsilon_{max}, \end{cases} \quad (1.1)$$

with,

$$\varepsilon_{max} = \frac{f_y}{E_s} - \frac{\frac{f_{ct}}{E_s A_s}}{1 + \sqrt{3.6M\varepsilon_{max}}} A_c, \quad M = \frac{A_c}{\sum \pi \phi} \quad (1.2)$$

where  $E_s$  is the elastic modulus of the steel,  $f_y$  is the steel yield stress,  $f_{ct}$  is the tensile concrete strength,  $\sigma_{s,av}$  and  $\sigma_{ct,av}$  are the average tensile stresses in the reinforcing steel and in the concrete, respectively,  $A_s$  is the cross section of the steel bars,  $\varepsilon_{s,av}$  is the average strain in the reinforcing bar and  $A_c$  is the area of concrete bonded to the bar participating in the tension stiffening effect.

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## References

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