Allee effects in random environments: impact on harvested populations and their sustainability

Carlos A. Braumann [♭△] [1], Clara Carlos[♮△] and Nuno M. Brites[♢]

(♭) Departamento de Matemática, Escola de Ciências e Tecnologia, Universidade de Évora, Rua Romão Ramalho 59, PT 7000-671 Évora, Portugal.

(♮) Escola Superior de Tecnologia do Barreiro, Instituto Politécnico de Setúbal, Rua Américo da Silva Marinho, PT 2839-001 Lavradio, Portugal.

(△) Centro de Investigação em Matemática e Aplicações, Instituto de Investigação e Formação Avançada, Universidade de Évora, Rua Romão Ramalho 59, PT 7000-671 Évora, Portugal.

(♢) ISEG – School of Economics and Management, Universidade de Lisboa & REM – Research in Economics and Mathematics, CEMAPRE, Rua do Quelhas 6, PT 1200-781 Lisboa, Portugal.

Abstract

The growth dynamics of a harvested population in a random environment can be described by the general stochastic differential equation model $\frac{dX(t)}{X(t)} = f(X(t))dt + \sigma(X(t))dW(t) - qE(t)dt$, where $f(X)$ is the (*per capita*) natural average growth rate, $\sigma(X(t))dW(t)$ is the deviation from average due to environmental fluctuations (with $W(t)$ a standard Wiener process and noise intensity $\sigma(X) > 0$) and $qE(t)$ is the mortality rate caused by harvesting (with $E(t) \geq 0$ the harvesting effort and $q > 0$ the catchability). Assuming autonomous efforts, $E(t) = E(X(t))$, we obtain results on extinction and on existence of a stochastic equilibrium for models with and without Allee effects, where f, E and σ are general functions satisfying only biologically dictated assumptions and mild regularity assumptions. This generalizes previous results in [1] (for models without Allee effects) and in [4] (for non-harvesting models with Allee effects and constant $\sigma(X) \equiv \sigma$).

The impact of Allee effects is assessed qualitatively for these general models and quantitatively for particular models, namely the logistic model (without Allee effects) and the logistic-like model with Allee effects, both with harvesting. We compare them in terms of times for realistic extinction and in terms of optimal harvesting efforts (some particular results can be seen in [5, 2] and [3]).

References

[1] Braumann, C. A., Variable effort harvesting models in random environments: Generalization to density-dependent noise intensities. *Mathematical Biosciences*, 177&178: 229–245, 2002.

[2] Brites, N. M., Braumann, C. A., Moments and probability density of threshold crossing times for populations in random environments under sustainable harvesting policies. *Computational Statistics*, `https://doi.org/10.1007/s00180-022-01237-0`, 2022.

[3] Brites, N. M., Braumann, C. A., Profit optimization of stochastically fluctuating populations under harvesting: The effects of Allee effects. *Optimization*, 71(11): 3277–3293, 2022.

[4] Carlos, C., Braumann, C. A., General population growth models with Allee effects in a random environment. *Ecological Complexity*, 30: 26–33, 2017.

[5] Carlos, C., Braumann, C. A., Filipe, P. A., Models of individual growth in a random environment: Study and application of first passage times. In: *Advances in Regression, Survival Analysis, Extreme Values, Markov Processes and Other Statistical Applications*, 103–111, Springer, Berlin Heidelberg, 2013.

[1] braumann@uevora.pt