Delay effects on a classical dryland vegetation model

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Abstract

Vegetation in semiarid areas exhibit spatial discontinuities and complex temporal dynamics. Different models have been proposed in the literature to describe the dynamic and spatial characteristic of semiarid vegetation. Klausmeier [1] proposed a model consisting in a system of two partial differential equations relating plant growth and soil water, showing that, under water limitation, increased infiltration of water by plants could produce characteristic spatial patterns of vegetation found in drylands. This classical model has been analysed and extended in later years in different aspects (e.g., [2, 3]). In [4], discrete delays were incorporated to account for the lag between water infiltration into the soil and the following water uptake by plants.

In this communication, we consider more ecologically realistic distributed delays for soil water availability. By considering Gamma distributed delay kernels, we use the so called *linear chain trick* [5, 6], which allows to convert a delay differential system into an ordinary differential system of higher dimension. We analyse the effects of different delay types on the stability and bifurcations of both mean field and spatial Klausmeier models. Numerical examples for realistic parameter values, for semiarid vegetation of both grasses and trees, are presented.

References

[1] Klausmeier, C.A., Regular and irregular patterns in semiarid vegetation, *Science*, 284: 1826–1828, 1999.

[2] van der Stelt, S., Doelman, A., Hek, G., Rademacher, J.D.M. Rise and fall of periodic patterns for a generalized Klausmeier–Gray–Scott Model. *Journal of Nonlinear Science*, 23: 39–95, 2013.

[3] Wang, X., Zhang, G. The influence of infiltration feedback on the characteristic of banded vegetation pattern on hillsides of semiarid area. *PLoS ONE*, 14:e0205715, 2019.

[4] Li J, Sun G-Q, Guo Z-G. Bifurcation analysis of an extended Klausmeier–Gray–Scott model with infiltration delay, *Studies in Applied Mathematics*, 148: 1519–1542, 2022.

[5] Cushing, J.M. *Integrodifferential Equations and Delay Models in Population Dynamics*. Springer, Heidelberg 1977.

[6] Smith, H. *An Introduction to Delay Differential Equations with Applications to the Life Sciences*. Springer, New York, 2011.

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