System decoupling: filters and phase synchronization

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Abstract

The quadratic systems decoupling problem is one with the longest tradition in mechanical and electrical engineering. Given a second-order linear dynamical system

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t), \tag{1.1}$$

its spectral structure is formed by the eigenvalues and corresponding multiplicities of its associated matrix polynomial $Ms^2 + Cs + K$, that is, by its elementary divisors. The decoupling problem is to obtain a transformation bringing system (1.1) to a new one

$$M_D\ddot{p}(t) + C_D\dot{p}(t) + K_Dp(t) = g(t) \tag{1.2}$$

with the same spectral structure such that M_D, C_D and K_D are diagonal matrices. In addition, it is wanted that the solutions $x(t)$ of the original system can be easily obtained from the solutions $p(t)$ of the diagonal one. The former system is said to be coupled and the latter decoupled. The process of transforming system (1.1) into system (1.2), when possible, is known as decoupling.

It is well-known that, in general, decoupling cannot be achieved by the usual change of variable $p(t) = Tx(t)$, with T nonsingular. In recent years two methods have been developed to effectively decouple quadratic systems. One of them is based on the notion of coprime filters (see [2]), and the other implements a technique called phase synchronization (see [3, 4]).

In this talk we show a connection between both methods. It has been helpful to have parametrized the set of filters connecting two isospectral systems (see [1]), because we have been able to identify the parameter that corresponds to phase synchronization.

References

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