Efficient multidimensional family of iterative methods free of Jacobian matrices

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Abstract

Solving systems of nonlinear equations is an increasingly frequent problem in any scientific application. Due to the difficulty of analytical calculations in finding the exact solution to such problems, it is common to use iterative fixed-point algorithms that approximate the solution of the systems. Also, as science progresses, the number of variables involved in the processes increases, so the dimensions of the systems of nonlinear equations that model engineering applications are becoming larger and larger. This makes it necessary to design efficient iterative schemes.

Different multidimensional iterative methods have been proposed in recent decades with the aim of accelerating the speed of convergence and improving computational efficiency [1]. However, the vast majority of them use Jacobian matrices and cannot be used for not differentiable functions or functions whose related Jacobian matrix does not have a known expression.

In this work, we propose the following family of iterative methods to approximate simple roots of systems of nonlinear equations $F(x) = 0$, being $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$:

$$\begin{aligned}
y^{(k)} &= x^{(k)} - \left[2x^{(k)} - x^{(k-1)}, x^{(k-1)}; F\right]^{-1} F(x^{(k)}) \\
z^{(k)} &= y^{(k)} - \beta \left[2x^{(k)} - x^{(k-1)}, x^{(k-1)}; F\right]^{-1} F(y^{(k)}) \qquad\qquad k = 0, 1, 2 \dots \\
x^{(k+1)} &= z^{(k)} - \frac{1}{\beta} \left[2x^{(k)} - x^{(k-1)}, x^{(k-1)}; F\right]^{-1} \left(-(\beta-1)^2 F(y^{(k)}) + F(z^{(k)})\right)
\end{aligned} \tag{1.1}$$

The iterative class (1.1) has order of convergence four for any value of parameter $\beta \in \mathbb{R}$, $\beta \neq 0$. In addition, it uses the previous and the current iterations to obtain the following approximation to the solution of the problem and, instead of a Jacobian matrix, its iterative structure computes the Kurchatov's divided difference operator [2]. In this paper, the proposed family is analysed in terms of convergence and its computational efficiency for solving high-dimensional nonlinear systems is also compared with other methods of similar characteristics, showing that good approximations to the solutions of the considered nonlinear problems are obtained.

References

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