Stability of TASE Runge-Kutta methods for parabolic PDEs

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Abstract

Recently, a new technique was introduced in [1] for the solution of a stiff IVP. Firstly, the ODE system is modified as follows:

$$\begin{cases} y_t = T_p(kJ) f\big(t, y(t)\big), \\ y(t_0) = u_0, \end{cases} \qquad T_p \in \mathcal{L}(\mathbb{R}^d, \mathbb{R}^d), \quad f : \mathbb{R} \times \mathbb{R}^d \to \mathbb{R}^d, \quad t \in [t_0, T]. \tag{1.1}$$

Then, an explicit RK method of order p is applied to the modified problem (1.1). The linear operator T_p, called TASE operator, is a function of the product kJ, where J is the Jacobian of f evaluated at the current time grid-point. T_p can also depend on one or more free real parameters to be set in such a way that the used explicit RK method stably and accurately solves the modified ODE system (1.1).

To preserve the order of the explicit RK method, the operator T_p must satisfy the property $T_p = I + O(k^p)$, where I is the identity matrix of order d. In fact, in this way, the exact solution of the perturbed problem (1.1) differs from that of the original one of $O(k^p)$. Therefore, applying a method of order p to the perturbed ODE system (1.1) leads to a numerical solution that approximates with order p the exact one of the original IVP, i.e. $||u(t_n) - y_n|| = O(k^p)$.

A new family of RK-TASE methods was developed by Montijano et al. in [2], calculating

$$T_p(kJ; \underline{\alpha}) = \sum_{j=1}^{p} \beta_j (I - \alpha_j kJ)^{-1}, \quad \underline{\alpha} \in \mathbb{R}^p, \qquad \beta_j = \Big(\frac{1}{\alpha_j}\Big)^{p-1} / \prod_{l \neq j} \Big(\frac{1}{\alpha_j} - \frac{1}{\alpha_l}\Big), \quad j = 1, \ldots, p. \tag{1.2}$$

The extended family of TASE operators (1.2) involves p free real parameters α_j, $j = 1, \ldots, p$. The values of the coefficients β_j, $j = 1, \ldots, p$, are a-priori fixed to get order p for the TASE operator, since the Taylor series expansion near zero of (1.2) is given by

$$T_p = I + K_p k^p J^p + O(k^{p+1}), \quad K_p = \sum_{j=1}^{p} \beta_j \alpha_j^p.$$

In this work, we will study the numerical stability of the proposed family of RK-TASE methods for the linear problem

$$y' = (A + B)y, \tag{1.3}$$

when A and B do not necessarily commute and $T_p(kA)$ depends only in A (not J the Jacobian), in a similar way as it was done in [3, 4]. This is a first step to analyze the stability for some particular nonlinear parabolic problems.

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