Computing Lyapunov exponents for the study of the dynamical behaviour of Chebyshev's method on polynomials

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Abstract

It is well known (see [3], for instance) that Chebyshev's method presents some interesting dynamical properties, like superattracting n-cycles. Some of these dynamical properties, such as the appearance of superattracting extraneous fixed points, are not present in other profusely studied root-finding methods like Newton's. The main focus of this work, framed in the context of polynomial root-finding algorithms, will be to give a numerical and graphical study of the dynamical behaviour of Chebyshev's method based on the computation of the Lyapunov exponents of the induced rational map.

In order to do this, we define a function (called a Lyapunov function) which depends on the spherical derivative of the rational map, and which is constant in each of its basins of attraction. Being able to compute these constants, closely related with the Lyapunov exponents associated with the rational map, can provide us useful information about the dynamics of the system.

The implementation in Julia Language (which can be found in [2]) of the algorithms that allows us to compute the Lyapunov exponents and the basins of attraction induced by a rational map solves some frequent computational problems, like overflows or indeterminations. This is achieved by considering the Hopf-endomorphism induced by the given rational map, and iterating it over the complex projective line $P^1(\mathbb{C})$. This approach was also undertaken in [1] for the study of the iteration of rational maps, with emphasis on the particular case of those induced by Newton's method when applied to cubic polynomials.

References

[1] V. Álvarez, J.M. García, L.J. Hernández and M.T. Rivas, *Algorithms for computing attraction basins of a rational self-map of the Hopf fibration based on Lyapunov exponents*, Preprint (2022).

[2] V. Álvarez-Aparicio, *Lyapunov Cycle Detector (LCD.jl)*, github.com/LCD, MIT license, v.1.4, 2022.

[3] J.M. Gutiérrez and J.L. Varona, *Superattracting extraneous fixed points and n-cycles for Chebyshev's method on cubic polynomials*, Qualitative Theory of Dynamical Systems, 19:1-23, 2020.

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