Constructing exact numerical solutions and nonstandard difference schemes for second order linear delay differential equations

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Abstract

Exact numerical solutions for delay differential equations have been previously obtained for first order scalar linear initial-value problems [1], and also for first order linear systems with commuting matrix coefficients [2]. These results were extended in [3] to the case of first order linear systems with non-commuting matrix coefficients, but the resulting expressions included infinite sums, that in general needed to be truncated to produce numerical approximations. These truncated sums were the basis to define nonstandard numerical schemes of arbitrary order, that were shown to possess good properties in preserving the asymptotic stability of the corresponding continuous solutions. It was also suggested in [3] that it could be possible to obtain closed-form exact numerical solutions, based on the infinite sums expressions given there, for particular problems depending on the specific structures of the coefficient matrices.

In this communication, we discuss how this approach can be made effective for some second order linear delay differential problems, by transforming them in the usual way to first order systems, in general with non-commuting matrix coefficients, and taking advantage of the simple structures of the resulting matrix coefficients. We obtain closed-form exact numerical solutions, with the use of some special functions, and derive from them high order nonstandard numerical schemes that are proven to be dynamically consistent with the asymptotic stability of the continuous solutions. Numerical examples are presented, illustrating the computation of exact numerical solutions, as well as the order convergence and asymptotic stability preserving properties of the nonstandard difference schemes.

References

[1] García, M.A.; Castro, M.A.; Martín, J.A.; Rodríguez, F. Exact and nonstandard numerical schemes for linear delay differential models. *Appl. Math. Comput.* **2018**, *338*, 337–345.

[2] García M.A.; Castro M.A.; Martín J.A.; Rodríguez F. Exact and nonstandard finite difference schemes for coupled linear delay differential systems. *Mathematics.* **2019**, *7*, 1038.

[3] Castro, M.A.; Sirvent, A.; Rodríguez, F. Nonstandard finite difference schemes for general linear delay differential systems. *Math. Meth. Appl. Sci.* **2021**, *44*, 3985–3999.

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