# Bancroft's GPS navigation solution: relativistic interpretation

Ramón Serrano Montesinos$^\flat$,[1] Juan Antonio Morales-Lladosa$^\flat$

($\flat$) Departament d'Astronomia i Astrofísica, Universitat de València,
Av. Vincent Andrés Estellés, Edifici Jeroni Muñoz, 46100 Burjassot, València, España.

## Abstract

In the context of Global Navigation Satellite Systems (GNSS), a modern approach is that of *relativistic* positioning. In Minkowski space-time, a relativistic positioning system (RPS) can be thought of as a set of at least four emitters $A$ $(A = 1, 2, 3, 4)$ of world-lines $\gamma_A(\tau^A)$, broadcasting their respective proper times $\tau^A$ by means of electromagnetic signals. In a RPS, the basic observable is the set of four proper times $\{\tau^A\}$ received at an event $x$ by the user. These are the user's emission coordinates.

Suppose the four world-lines $\gamma_A(\tau^A)$ are known in an inertial coordinate system $\{x^\alpha\}_{\alpha=0}^3$, then the location problem consists in finding the transformation between the user's emission coordinates and its inertial coordinates in that coordinate system, by solving the following algebraic system of four non-linear equations: $(x - \gamma_A)^2 = 0$, $(A = 1, 2, 3, 4)$, where $x$ is the sought user's position four-vector with respect to $\{x^\alpha\}$. The solution to these equations, mapping the user's emission coordinates to its inertial coordinates, is what we may call the RPS coordinate transformation or the RPS solution [1].

The purpose of this communication is to bring the (non-relativistic) theoretical foundations of current GNSSs closer to the RPS approach, by recovering from the RPS coordinate transformation equation one of the classical solutions to the problem which is still in use today: Bancroft's closed-form solution (with four emitters) [2].

## References

[1] Coll, B., Ferrando, J. J., and Morales-Lladosa, J. A., Positioning Systems in Minkowski spacetime: from emission to inertial coordinates, *Class. Quantum Grav.*, Volume(27): 065013 (17pp), 2010.

[2] Bancroft, S., An algebraic solution of the GPS equations, *IEEE Transactions on Aerospace and Electronic Systems*, Volume(AES-21): 56–59, 1985.

[1]rasemon@alumni.uv.es