Dynamical systems and entropy: state of research

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Abstract

The objective of the article is to introduce the entropy and the temperature in the context of abstract dynamical systems. Then, I present the state of research of this objective. Concretely, let $q_k(t)$, k=1, 2, ..., n, be the abstract variables of a dynamical system, with $\boldsymbol{q} = (q_1, q_2, \ldots, q_n)$, with its Dirac's Hamiltonian [1] :

$$\dot{q}_k(t) = f_k(t, \boldsymbol{q}) \ ; \ k\text{=1, 2, ..., } n \tag{1}$$

$$H(t, \boldsymbol{q}, \boldsymbol{p}) = \sum_{j=1}^{n} f_j(t, \boldsymbol{q}) \cdot p_j - \sum_{j=1}^{n} f_j(t, \boldsymbol{q}) \cdot g_j(t, \boldsymbol{q}) + h(t, \boldsymbol{q}) \tag{2}$$

In (2) p_j are the canonical moments, and the functions $g_j(t, \boldsymbol{q})$ and $h(t, \boldsymbol{q})$ are given by equations presented in [2]. However, the hypotheses stated in [2] to introduce the entropy $\mathrm{S}(t, \boldsymbol{q})$ and the temperature $\mathrm{T}(t, \boldsymbol{q})$ do not drive to any concluding result. Therefore, the approach must be changed by stating different hypotheses: 1. By introducing a new equation as the entropy time derivative $\dot{\mathrm{S}}(t, \boldsymbol{q})$; 2. Following hamiltonian Dirac's approach, the system dimension n of (1) must be even, then, if n is odd, considering $\boldsymbol{q} = (q_1, q_2, \ldots, q_n, S)$, $N = n + 1$, the $\dot{q}_N = \dot{\mathrm{S}}(t, \boldsymbol{q}) = f_N(t, \boldsymbol{q}, S)$ equation is added, and if n is even, considering $\boldsymbol{q} = (q_1, q_2, \ldots, q_n, q_{n+1}, S)$, $N = n + 2$, the $\dot{q}_{n+1} = 0$ and $\dot{q}_N = \dot{\mathrm{S}}(t, \boldsymbol{q}) = f_N(t, \boldsymbol{q}, S)$ equations are added; 3. Eq. (2) is simplified by considering that $h(t, \boldsymbol{q}) = \sum_{j=1}^{N} f_j(t, \boldsymbol{q}) \cdot g_j(t, \boldsymbol{q})$, that is:

$$H(t, \boldsymbol{q}, \boldsymbol{p}) = \sum_{j=1}^{N} f_j(t, \boldsymbol{q}) \cdot p_j = \sum_{j=1}^{N-1} f_j(t, \boldsymbol{q}) \cdot p_j + f_N(t, \boldsymbol{q}, S) \cdot p_N \tag{3}$$

I assume in (3) that the Hamiltonian is the corresponding generalized internal energy of the dynamical system. Then, the temperature is defined as in the Thermodynamics context $\mathrm{T}(t, \boldsymbol{q}) = \frac{\partial H}{\partial S} = \frac{\partial f_N(t, \boldsymbol{q}, S)}{\partial S} p_N$.

In addition, the equations for the $g_j(t, \boldsymbol{q})$ functions become from those presented in [2] as:

$$\frac{\partial g_j(t,\boldsymbol{q})}{\partial t} + \sum_{l=1}^{N} \frac{\partial g_j(t,\boldsymbol{q})}{\partial q_l} f_l(t, \boldsymbol{q}) = -\sum_{l=1}^{N} \frac{\partial f_l(t,\boldsymbol{q})}{\partial q_j} g_l(t, \boldsymbol{q}) \ ; \ j = 1, 2, \ldots, N \tag{3}$$

Moreover, applying a Gibbs-Duhem generalized equation, it provides a new equation, which coincides with the differential form for the generalized internal energy, that is:

$$g_N(t, \boldsymbol{q}) \sum_{l=1}^{N} \frac{\partial f_N(t,\boldsymbol{q},S)}{\partial q_l} f_l(t, \boldsymbol{q}) = -\sum_{j=1}^{N-1} \sum_{l=1}^{N-1} \frac{\partial f_j(t,\boldsymbol{q})}{\partial q_l} g_j(t, \boldsymbol{q}) \cdot f_l(t, \boldsymbol{q}) \tag{4}$$

The internal energy of the classical reversible Thermodynamics is deduced. Thus, good way.

References

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