Delay formulation of a model of hierarchically dependent individual growth rate

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Abstract

The usual tool for formulating dynamics models for populations structured by physiological variables (age, size, phenotype,...) are non-linear hyperbolic pdes with non-local terms for the density with respect to the structuring variable.
An alternative is to formulate the models using the integral renewal equations one obtains from integration along characteristic lines. In this setting the state variables usually are the birth rate and an accompanying variable for the environmental condition (for instance the density of resources). The renewal equation is interpreted as a rule for extending a function of time toward the future on the basis of the (assumed to be) known history of the state variables, so as a delay equation. By shifting along the extended function, i.e. by updating the history one obtains a dynamical system ([1]).
An advantage of the second option is that the resulting solution semigroup is differentiable under suitable hypotheses on the model ingredients whereas the same is not true in the pde formulation, the reason being that the initial population density is translated, by growth, over a variable distance on the structuring space. And if the initial density is not absolutely continuous, then there is no differentiable dependence on this distance.
As a result of using the delay formulation, rigorous linearization results (stability and instability for instance) are obtained which in turn are only formal in the case of the corresponding quasilinear pdes ([3]).
We will see an example of hierarchical competition in which the analysis allows a fairly complete description of the dynamics. Although the assumption of hierarchical competition leads in principle to an infinite dimensional environmental variable, the simplifying assumption of an individual growth depending only on this variable allows a drastic reduction to a scalar delay equation for the birth rate. The main technical difficulty in the analysis comes then the proof of differentiability of the function defining the model delay equation ([2]).

References

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