Pathwise methods for a class of stochastic models with non-globally Lipschitz coefficients

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Abstract

We propose an approach for the construction of numerical integrators for multiplicative-noise stochastic differential equations with non-globally Lipschitz continuous coefficients. For this, me devise an appropriate invertible continuous transformation, linking the solution to the stochastic equation to the solution of an auxiliary random differential equation (RDE) that has the Ornstein-Uhlenbeck process as the only input parameter of the system. In this way, based on this explicit conjugacy between both equations, new pathwise numerical schemes are constructed which outperform other integrators in the literature. In particular, by applying this approach, we propose an exponential-based method for a compartmental epidemic SVIR model, with non-globally Lipschitz coefficients, that describes a continuous vaccination strategy with environmental noise effects. A simulation study is presented to illustrate the practical performance of the introduced methods and a comparative analysis with other commonly used integrators in applications is carried out.

References

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