Increase of the convergence order of iterative methods of order ρ to $\rho + 3$ for equations and systems of equations

Alicia Cordero $^{\flat}$, Miguel A. Leonardo Sepúlveda$^{\natural 1}$,María P. Vassileva $^{\natural}$ and Juan R. Torregrosa$^{\flat}$

($\flat$) Universitat Politècnica de València,
Instituto de Matemática Multidisciplinar, Universitat Politècnica de València, Camino de Vera, s/n, 46022 Valencia, Spain.
($\natural$) Instituto Tecnológico de Santo Domingo,
Ciencias Básicas y Ambientales (CBA), Instituto Tecnológico de Santo Domingo (INTEC), Area de Ciencia Básica y Ambiental, Av. Los Próceres, Gala, Apartado postal 342-9 and 249-2, Santo Domingo 10602, Dominican Republic.

Abstract

The resolution of equations and systems of nonlinear equations is positioned among the most important problems in the numerical field, both from the theoretical and practical point of view of applied mathematics, as well as from many branches of science, engineering, physics, computer science, astronomy, finance, among others. The convergence order of an iterative method is extremely important for these purposes, since it allows the succession of iterates to converge to the system solution more quickly, and if the convergence order is even higher.

In our work we present a technique that allows to increase the convergence order from to units using Newton's method as a predictive step in addition to certain conditions that must be met, this procedure is applied to iterative methods to solve non-linear equations as well as systems of equations.

The main idea is to add to some of the families of known methods, such as King's family, Ostrowski's method, Chun, among other order methods, a third step that we propose in our work and thus obtain a new method that increases the order of convergence of said method by three units, whether for the scalar or vector case.

Applying this third step to any iterative method of order three, four, five,..., ; we obtain new order methods six, seven, eight,...,, respectively.

In addition, we compare the dynamic planes obtained with King's family presented by Cordero, Alicia, et al., "Chaos in King's iterative family", whose work shows an unstable behavior for certain values of the parameters of the family and then we obtain the same dynamic planes for the same values of the parameters with our method increased in the order of convergence seven and we observe that

Finally, we solve some academic problems of systems of nonlinear equations and obtain the solution of Fisher's equation, a non-linear diffusion EDP, discretizing it by the finite difference method and finally solving it numerically, thus showing how efficient the methods obtained in our work turn out to be.

Best regards,

[1]miguel.leonardo@intec.edu.do

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