Novel optimal fourth-order methods with perturbed derivatives for solving nonlinear equations with multiplicity

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Abstract

The first and most striking feature of this contribution is that we have developed a multipoint optimal general class of fourth-order methods that will converge even if the guess is far from the root or the derivative is small in the vicinity of the required root. In addition, our scheme do not require to calculate the mth power of a function or it's derivative $\left[e.g.\ \left(\frac{f(x_n)}{f'(x_n)}\right)^{\frac{1}{m}}\right]$ (required nowadays in multiple root methods). In this approach, we reduce the computational cost. The development of the scheme is stand on two variable weight functions. Li et al. scheme and the Zhou et al. family of methods are obtained as special cases of our proposed scheme. If, at any point during the search of required root, $f'(x) = 0$, Newton's method and its variants fail miserably due to division by zero. Our methods do not exhibit this type of behavior. We use numerous real-world scenarios to demonstrate the applicability of our approaches and to compare them to earlier iterative methods. Our methods outperform the existing ones in terms of CPU timing, absolute residual errors, asymptotic error constants, absolute error difference between two consecutive iterations, and approximated roots, according to the computational results we have obtained. The theoretical findings are also supported by dynamical planes.

Keywords: Nonlinear equations. Multiple roots. Newton's method. Optimal order of convergence. Efficiency index.