A highly efficient class of optimal fourth-order methods for nonlinear systems

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Abstract

We present a new class of highly efficient two-parameter optimal iterative methods for nonlinear equation systems that generalizes Ostroski's method, King's family, Chun's method and KLAM family in vectorial case. This last class is an extension to systems of the Ermakov's Hyperfamily in [6]. Kung y Traub in [2] established a conjecture on optimality for the scalar case. Analogously for systems, the Cordero-Torregrosa's conjecture, in [5], states that the order of convergence of any multistep method for nonlinear systems cannot exceed $2^{k_1+k_2-1}$ *(the optimal order)*, where k_1 and k_2 are the number of evaluations of the Jacobian matrix and the nonlinear function per iteration, respectively, and $k_1 \leq k_2$. Up to the present the only optimal method was Newton of order 2. Our contribution is the only class of optimal fourth-order methods for systems known so far. And, as if that were not enough, also with an infimum computational cost, if compared to any method of order greater than 3, since the weights used are scalar. Through the convergence analysis the fourth-order is demonstrated. The high efficiency of the class is studied, compared with methods of the same order, even higher. Some numerical proofs are presented and we analize it robustness, showing that it has great stability.

References

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