An efficient multidimensional Liouville equation solver and its applications to random differential equations

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Abstract

The Liouville, Liouville-Gibbs, or Continuity Equation is a classical partial differential equation (PDE) describing the conservation of a certain quantity under the dynamics of a vector field; that is, it is a conservation law [1]. In mathematics, the Liouville equation can be used to describe the conservation of probability. It appears when studying ensembles of trajectories, for example, in optimal transportation theory or random/stochastic differential equations[2, 3].

Specifically, we use the Liouville Equation to simulate the forward-in-time evolution of the probability density function of stochastic processes that verify a random differential equation[4, 5]. This approach is important because it allows us to simulate its evolution even when no closed-form solution is available.

This contribution will present our novel computational approach to approximate the Liouville equation's solution and obtain important statistical information such as confidence regions or computing probabilities of specific events.

References

[1] Evans, L.C., Partial differential equations. Providence, R.I., American Mathematical Society, 2010.

[2] Santambrogio, F., Optimal Transport for Applied Mathematicians. Calculus of variations, PDEs and Modeling. Birkhäuser Cham, 2015.

[3] Soong, T.T., Random Differential Equations in Science and Engineering. New York, Academic Press, 1973.

[4] Bevia, V.J., Cortés, J.C., Villanueva, R.J., Forward uncertainty quantification in random differential equation systems with delta-impulsive terms: Theoretical study and applications *Mathematical Methods in the Applied Sciences*, 1-21, 2023. DOI: 10.1002/mma.9226

[5] Bevia, V.J., Cortés, J.C., Jornet, M., Villanueva, R.J., Probabilistic analysis of a general class of nonlinear random differential equations with state-dependent impulsive terms via probability density functions *Communications in Nonlinear Science and Numerical Simulation*, 119:107097, 2023.

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