Tensor eigenvector centrality for higher order networks: uniform vs. non-uniform case

G. Contreras-Aso ♭ ♮, C. Pérez-Corral♮ and M. Romance♭ ♮,[1]

(♭) Department of Applied Mathematics, Science and Engineering of Materials, and Electronic Technology, ESCET, Universidad Rey Juan Carlos, Móstoles (Madrid, Spain).

(♮) Laboratory of Mathematical Computation on Complex Networks and their Applications, Universidad Rey Juan Carlos, Móstoles (Madrid, Spain).

Abstract

Measuring the relevance of nodes and edges in a network is a critical aspect of Network Science [5] with a wide range of applications, including identifying influential individuals in social networks, ranking web pages, identifying impactful scientific journals and many more. Spectral centralities are particularly important centrality measures that arise from solving an eigenvalue problem, where the importance of nodes is determined by the entries of the dominant eigenvector of a matrix associated with the graph, such as the classic eigenvector centrality [4] of a strongly connected network $G = (V, E)$ which is given by a normalized and positive eigenvector associated with the spectral radius of the adjacency matrix of G.

However, there is a fundamental limitation in the network representation of a distributed system, since networks capture only pairwise interactions, while many real-world systems involve group interactions among their nodes [1, 3]. Simplicial complexes and hypergraphs are natural alternatives to describe such systems and in recent years, there has been a surge of interest in these representations, which has revolutionized our ability to tackle real-world systems characterized by more than simple dyadic connections [3].

In this talk we will present some extensions of the notion of graph eigenvector centrality to hypergraphs. While the existence and uniqueness of the classic graph eigenvector centrality is given by the Perron-Frobenius theorem, the hypergraphs extensions of such centrality measure are analyzed in terms of Perron-Frobenius theory for hypermatrices related either to uniform and non-uniform hypergraphs [2]. In addition, several numerical comparisons are presented for different real-life systems modelled with higher order interactions.

References

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[1] miguel.romance@urjc.es