On Approximate Inverse LU Preconditioning

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Abstract

Preconditioning is a standard technique used in numerical linear algebra to solve a large, sparse nonsymmetric linear system $Ax = b$, where A is a nonsingular matrix using iterative methods. The main goal of it is to transform the initial linear system into a new equivalent one which can be solved more efficiently. Left, right or two side preconditioning are different variants of this technique. For instance, left preconditioning consists in solving iteratively the linear system $M^{-1}Ay = M^{-1}b$ where the nonsingular matrix M is called the preconditioner. The preconditioner M must satisfy some a priori conditions: it should approximate the matrix A in some sense and its application must be as cheap as possible from a computational point of view. If M^{-1} is not explicitely available its application involves the solution a linear system with M at the preconditioning step. By contrast, approximate inverse preconditioners store explicitely the matrix M^{-1} and its application through matrix by vector has multiple advantages in numerical computation.

In this work we study factorized approximate inverse LU preconditioners that compute explicitely an approximation of A^{-1}. We use the Sherman–Morrison formula to obtain a decomposition of A^{-1}. The main difference with respect to the AISM preconditioner [1], is the way of applying recursively the inversion formula. Here to construct the new preconditioner we have a multiplicative representation of A instead of an additive one. Then we can use a compact representation of this decomposition to build the factorization of A^{-1} that we call V–AISM. We also show that dropping small nonzero elements during the computation of the preconditioner is breakdown free for M- and H-matrices. Moreover, the results of numerical experiments show that this new preconditioner is efficient and faster than AISM and other approximate inverse preconditioners that appear in the bibliography.

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References

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