Accurate bidiagonal decomposition of Lupaş-type (p,q)-analogue of the Bernstein basis and applications

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The collocation matrices of the Lupaş-type (p,q)-analogue of the Bernstein basis ((p,q)-Lupaş matrices in the sequel) are a generalization of the Vandermonde matrices obtained when replacing the monomial basis by a generalization of the Bernstein basis introduced in [1] and used in the area of CAGD: the Lupaş-type (p,q)-analogue of the Bernstein basis.

In this work we present an algorithm to compute the bidiagonal decomposition of these collocation matrices. The algorithm is based on results on total positivity and Neville elimination, and the explicit expressions obtained for the determinants involved in the process make it both fast and accurate [3]. Starting from this bidiagonal decomposition, and using also some of the algorithms of Koev [2], the accurate and efficient solution of several linear algebra problems involving (p,q)-Lupaş matrices is addressed: linear system solving, eigenvalue and singular value computation, and computation of the inverse and the Moore-Penrose inverse. The numerical experiments carried out show the good behaviour of the proposed approach, which gives very accurate results even when the condition number of the (p,q)-Lupaş matrices is very high.

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