Pricing Forward Freight Agreement contracts with time-delay stochastic processes

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1 Introduction

The shipping industry is essential for the global distribution of raw materials and commodities and plays an important role in the world economy, in fact, about the 90% of the world trade is carried by sea [11]. As a consequence, the market for ocean-going shipping freight has undergone a fundamental transformation from a service market, where the freight rate was simply viewed as the cost of transporting raw materials by sea, to a market where the freight rate can be bought and sold for investment purposes like any other financial asset or commodity. The high volatility in the freight market has led to a corresponding growth in the derivatives market of freights. Traditionally, this market has been used by players in the physical freight market to hedge their freight risks, though this changed rapidly with the increasing participation of investment banks and hedge funds. Market participants trade forward contracts on shipping freight rates, known as Forward Freight Agreements (FFA). These contracts are negotiated over-the-counter (OTC) and subsequently cleared through a clearing house and belong to the wider family of average-price forward contracts.

In Finance, it is commonly assumed that the asset's present prices reflect all necessary information and that its history is not relevant. However there is some empirical evidence which shows that the past returns are important and affect to the present ones, see [2] for stock prices or [9] for commodities. Consequently, some scholars have considered time-delay stochastic processes to price financial derivatives, such as options (see [1]), bonds (see [3]) and commodity futures (see [5]).

The aim of this paper is to consider the effect of past spot freight rates on FFA prices. In particular, we assume that the spot freight rate verifies a stochastic delay differential equation (SDDE) and compare the obtained FFA prices with those in the Panamax market.

2 The model

We assume that $(\Omega, \mathcal{F}, \mathcal{P})$ is a given probability space equipped with a filtration $\{\mathcal{F}_t\}_{0\leq t\leq T}$ satisfying the usual conditions, see [10]. We consider that the spot freight rate follows a stochastic delay

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diffusion process, under the equivalent risk-neutral measure ($\mathcal{Q}$-measure)

$$\begin{aligned} &dS(t) = \mu(S(t), S(t-d))dt + \sigma(S(t), S(t-d))dW^{\mathcal{Q}}, \quad t \in (0,T], \\ &S(t) = \phi(t), \quad t \in [-d, 0], \end{aligned}$$

where d is a positive constant, a fixed delay, which is incorporated in the drift and volatility terms in order to take into account the past events on the current and futures states. The drift and volatility functions, μ and σ are deterministic continuous functions, satisfying suitable regularity conditions, see [10]. These functions depend on the spot freight rate in two instants of time. Additionally, $W^{\mathcal{Q}}$ is a Wiener process and ϕ is a deterministic continuous function, $\phi \in \mathcal{C}([-d, 0], \mathbb{R})$ which represents previous index information.

A FFA is a cash-settle financial contract which gives its owner the average spot price over the settlement period $[T_1, T_N]$, with $T_1 < T_2 < \ldots < T_N$, where T_1 and T_N are, respectively, the first and last trading days in a month. We denote by $F(t, S; T_1, \ldots, T_N)$ the FFA price. As the cost of entering the FFA is zero,

$$E^{\mathcal{Q}}\left[e^{-r(T_N-t)}\left(\frac{1}{N}\sum_{i=1}^{N} S(T_i) - F(t, S; T_1, \ldots, T_N)\right)|S(t) = S\right] = 0,$$

where $E^{\mathcal{Q}}$ is the conditional expectation under the $\mathcal{Q}$-measure.

Therefore, the FFA price at time t with settlement period $T_1, \ldots, T_N$, can be expressed as

$$F(t, S, ; T_1, \ldots, T_N) = \frac{1}{N}\sum_{i=1}^{N} E^{\mathcal{Q}}[S(T_i)|\mathcal{F}_t], \tag{1}$$

see [4] for more details.

In this work, we assume, as usual in the literature, that the time between two consecutive trading days is constant. That is, $T_{i+1} - T_i = \Delta$, $i = 1, \ldots, N-1$.

In the literature, there are some very-well known models for pricing FFA contracts, for example, in [8] the authors assume that the spot freight rate follows a geometric Brownian motion:

$$dS = \alpha^{\mathcal{Q}} S dt + \sigma S dW^{\mathcal{Q}}.$$

We call this model **Koekebaker *et al.*** This fact yields a closed-form solution for the FFA price given by:

$$F(t, S; T_1, \ldots, T_N) = S\frac{e^{\alpha^{\mathcal{Q}}(T_N-t)}}{N}\frac{e^{-\alpha^{\mathcal{Q}}N\Delta} - 1}{e^{-\alpha^{\mathcal{Q}}\Delta} - 1}. \tag{2}$$

In order to incorporate the past information in the spot freight rate, we consider these two time-delay stochastic processes, which have been previously used for pricing gold futures, see [5]:

$$dS(t) = \left(\mu_1 - \lambda_1\sigma_1 S(t-d)\right) S dt + \sigma_1 S(t-d) S dW^{\mathcal{Q}},$$

and considering mean reversion,

$$dS(t) = \left(k(\mu_2 - S) - \lambda_2\sigma_2 S(t-d)\right) dt + \sigma_2 S(t-d) dW^{\mathcal{Q}},$$

where μ_1, λ_1, σ_1, k, μ_2, λ_2 and σ_2 are constants. We call these models **Model1-delay** an **Model2-delay**, respectively.

The prices of the FFA contracts with these models can be calculated as an average of futures prices, see (1) and [4]. That is,

$$F(t, S; T_1, \ldots, T_N) = \frac{1}{N}\sum_{i=1}^{N} \hat{F}(t, S; T_i),$$

where $\hat{F}(t, S; T_i)$ is the price of a futures contract with maturity T_i, $i = 1, \ldots, N$. Then, taking into account the futures prices expressions for these models, which were previously obtained in [5], we obtain that the FFA price for the first model (Mod1-delay) is:

$$F(t, S; T_1, \ldots, T_N) = \frac{S\mathrm{e}^{-\mu_1 t}}{N} \sum_{i=1}^{N} \mathrm{e}^{\mu_1 T_i - \lambda_1 \sigma_1 \int_t^{T_1+(i-1)\Delta} S(z-d)\, dz}, \tag{3}$$

and for the second model (Mod2-delay):

$$\begin{aligned} F(t, S; T_1, \ldots, T_N) &= \frac{S-\mu_2}{N}\, \mathrm{e}^{-k(T_N-t)}\, \frac{\mathrm{e}^{kN\Delta}-1}{\mathrm{e}^{k\Delta}-1} + \mu_2 \\ &- \frac{\lambda_2 \sigma_2}{N} \sum_{i=1}^{N} \int_t^{T_i} S(z-d)\, \mathrm{e}^{-k(T_i-z)}\, dz. \end{aligned} \tag{4}$$

Sometimes, parametric models provide very unrealistic models because they assume restrictive functions for the spot freight rate stochastic process in order to obtain closed-form solutions for the FFA prices. Therefore, in this paper, in order to avoid imposing arbitrary parametric restrictions, we also use nonparametric techniques, such as the kernel method, to estimate the functions of the spot freight rate stochastic process. Note, that in this case, closed form solutions are not known, but numerical methods must be obtained to obtain approximated prices.

3 Empirical application

In this section, we show the supremacy of considering a time-delay stochastic process to model the spot freight rate when pricing FFA contracts using data from the Panamax market. This market is very volatile, flexible and broad with a relatively open structure. The importance of this market is also remarkable as the Baltic Panamax Index (BPI) contributes to the Baltic Dry index (BDI) with a weighting of 30%, see [7].

Our data consist on BPI (spot freight rate) observations from November 2014 to February 2023 and FFA contract prices with maturities from 1 to 24 months from December 2015 to February 2023. We divide these data into different blocks: the first two months are the delay period (November 2014-December 2014), the following 8 years are used to estimate the different functions (in-sample period: January 2015-January 2023) and finally, we keep the last year (out-of-sample period: February 2023-January 2024) to analyse the robustness of the models with delay when pricing FFA contracts.

On the one hand, we estimate the parameters of the Koekebaker *et al.* model, Model1-delay and Model2-delay. To this end, we use the futures prices (2), (3) and (4), the least squares method, BPI observations and FFA prices from the in-sample period. Moreover, for the time-delay parametric models, (3) and (4), we also consider the BPI in the delayed period. On the other hand, we estimate the drift and volatility of the spot freight rate stochastic process under $\mathcal{Q}$-measure (with and without delay) using the Nadaraya-Watson estimator with a Gaussian kernel, see [6] and [5].

In order to price FFA contracts in the out-of-sample period, we use (2), (3) and (4) for the parametric models. However, we use MonteCarlo method with the antithetic variable for the nonparametric models as a closed-form solution cannot be obtained. The errors are calculated with the root mean square error percentage error (RMSPE) for the different maturities of the available FFA contracts:

$$RMSPE_\tau = \sqrt{\frac{1}{N} \sum_{i=1}^{n} \frac{F_{i\tau}^{\theta} - F_{i\tau}^{M}}{F_{i\tau}^{M}}},$$

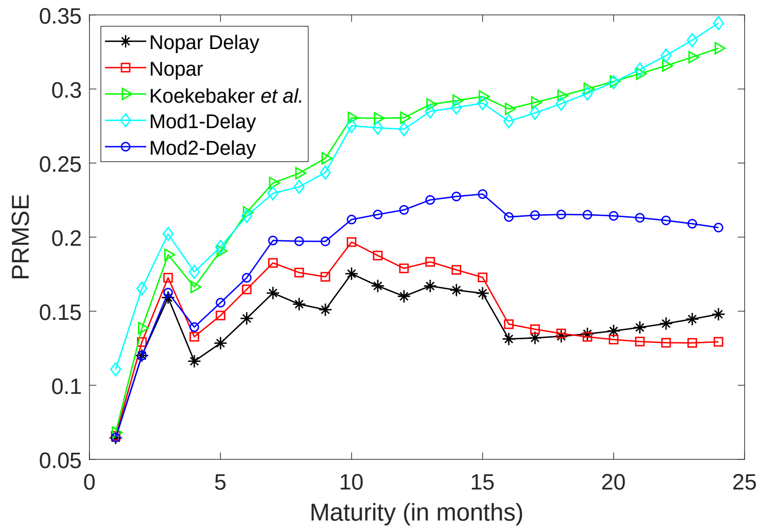

Figure 1: Root mean percentage squared error of the FFA contract prices for the in-sample period (January 2018-January 2023)

where $F_{i\tau}^{\theta}$ and $F_{i\tau}^{M}$ are the estimated and market prices, respectively, for each instant of time i, maturity τ, and N the number of observations for each maturity.

Figures 1 and 2 show the RMSPE for the different models and maturities for the in-sample and out-of-sample periods, respectively. In these graphs, we observe that the models with memory and/or estimated with nonparametric techniques provide the lowest errors. Furthermore, we think that the nonparametric models could even provide lower errors using a difference method to solve the pricing equation instead of the MonteCarlo method.

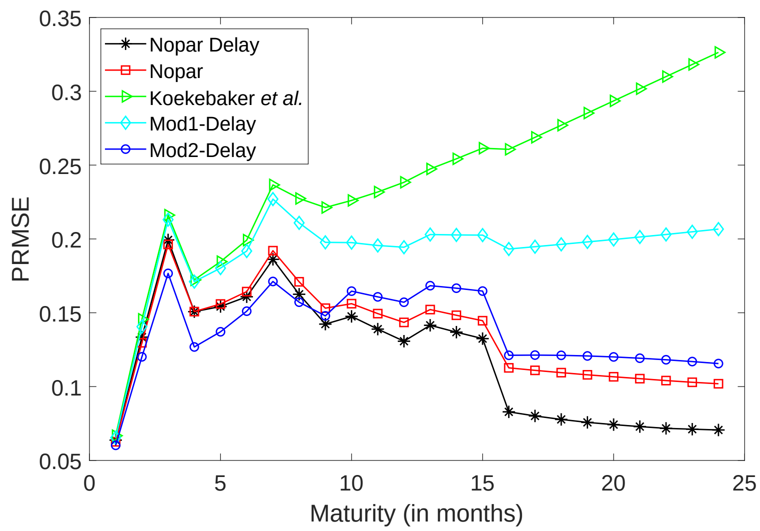

Figure 2: Root mean percentage squared error of the FFA contract prices for the out-of-sample period (February 2023-January 2024)

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