A spectral approach to Kemeny's constant

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Abstract

Kemeny's constant, a fundamental parameter in the theory of Markov chains, has recently received significant attention within the graph theory community. Originally defined for a discrete, finite, time-homogeneous, and irreducible Markov chain based on its stationary vector and mean first passage times, Kemeny's constant finds special relevance in the study of random walks on graphs. Kemeny's constant gives a measure of how quickly a random walker can move around a graph and is thus a good measure of the connectivity of a graph.

Several approximations and bounds for Kemeny's constant have been studied in the literature, see e.g. [1, 2, 4]. In this work, we continue this line of research by proposing several approximations and bounds of Kemeny's constant, which we derive using spectral techniques. In order to obtain our approximations we use the celebrated sparsification lemma by Spielman [3]. Furthermore, we evaluate Kemeny's constant, our approximations and its upper bound for several graph classes.

It is natural to study how graph structure informs a graph invariant. Spectral graph theory seeks to associate a matrix with a graph and to deduce properties of the graph from the eigenvalues (spectrum). While it is known that Kemeny's constant can be written in terms of the eigenvalues of some associated matrix to a graph, it would also be useful to approximate Kemeny's constant using fewer eigenvalues instead of the whole spectrum. This is the main focus of the second part of this work, where we explore how graph substructures and eigenvalues provide insights into Kemeny's constant.

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