

# Sustainability of a romantic relationship via computational differential games modeling

Jorge Herrera de la Cruz<sup>b</sup> and José-Manuel Rey<sup>a,1</sup>

(b) Department of Mathematics and Data Science, Universidad CEU San Pablo,  
28003 Madrid, Spain.

(a) Department of Economic Analysis, Universidad Complutense de Madrid,  
28223 Madrid, Spain.

## Abstract

Long-term romantic relationships are a fundamental pillar for the well-functioning of modern societies and the happiness of individuals [1]. However, understanding what makes a durable and rewarding relationship remains a challenging problem in the social sciences [2]. Part of the difficulty in addressing the problem lies in the scarcity of extensive observational data on marital quality or relationship feeling over time. In previous works we formulate a computational differential game to model romantic relationships designed to last forever, both in a deterministic and stochastic environment, see [3] and [4]. Our modeling allows us to synthesize temporal feeling trajectories as long as necessary.

We have recently shown that the feeling trajectories calibrated from our formalism render good approximations for real data obtained from monitoring couples over many years [5]. In this contribution, we use our calibrated model to analyze different issues regarding the sustainability of the relationship in the long term. Specifically, we first analyze the existence of a stationary feeling solution in the deterministic formulation of the model, and its dynamical instability. Then we show that the successful path towards stationarity is always demanding, in terms of the effort-making partners must exert to sustain the relationship. This is a key result, called the effort gap effect in the control-theoretic formulation of the problem [6]. The effort gap is a source of potential instability for the survival of the relationship in the long term. In the paper, we use a computational version of the model and a suitable algorithm to estimate the size of the effort gap at equilibrium for different types of relationships. We are particularly interested in the effect size with respect to the asymmetric efficiency of partners in effort-making. We also analyze the impact of the asymmetry on the stationary feeling and total happiness of the relationship.

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<sup>1</sup>j-man@ccee.ucm.es

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