Optimal properties of B-bases

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1 Introduction

A matrix whose minors are all nonnegative is called totally positive. Totally positive matrices have applications to many fields such as Combinatorics, Approximation Theory, Mechanics, Computer Aided Geometric Design, Statistics, Differential Equations or Economy (cf. [1, 8, 9, 10, 14].

A basis of a space U of univariate functions is called totally positive if all its collocation matrices are totally positive. If U has a totally positive basis, then there exists a totally positive basis of U (called B-basis) such that it generates all totally positive bases of U by means of totally positive matrices (see [3]). A first example of B-basis is the Bernstein basis of polynomials on a compact interval (see [2]), which is also a normalized basis, in the sense that the basis functions form a partition of the unity. In Computer Aided Geometric Design, normalized B-bases are the bases with optimal shape preserving properties. Optimal conditioning of the collocation matrices of the Bernstein basis basis among normalized totally positive bases was proved in [3] and the result was extended to normalized B-bases in [7]. Progressive iterative approximation is a useful method for curves fitting. It converges for normalized totally positive bases and in [5] it was proved that normalized B-bases provide the fastest convergence

Other optimal properties have been obtained for B-bases (such as optimal conditioning for the evaluation [11]) and their collocation matrices as well as for other related matrices (see [6] and [4]). We present some new contributions to this field, in particular some results related to optimal conditioning of Wronskian matrices of B-bases (see [13]) and with the fastest convergence of modified Richardson method (see [12]).

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