On the study of PN1 cross terms in the evolution of EMRIs

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1 Introduction

The problem of sources of gravitational waves, including the capture of a compact object by a supermassive black hole (an "extreme-mass ratio inspiral, EMRI"), has been treated in relativistic astrophysics with a post-Newtonian approach from the very beginning (see for instance [1] for a more detailed description). However, this approximation is not valid outside the two-body problem. In an astrophysical scenario, a binary emitting gravitational waves is very often perturbed by other forces. This problem is solved to first order using the cross terms of the post-Newtonian corrections [2]. In this paper we show the first results and what the implications are for the detection of an EMRI.

2 Methods

We use an N-body code (see [3] for a more detailed description of the numerical algorithms used) to numerically implement the computations of the PN1 corrections to Newtonian orbits including the cross terms. Suitable Black Holes are simulated orbiting around a central EMRI. Although this code is designed for Keplerian orbits, it is appropriately working with our PN1 corrections. Nevertheless, when increasing the number of stellar bodies it may take excessive computational time. So, other alternative algorithms should be used. AR-CHAIN code (see [4] for a description of this approach) is a promising one as it incorporates Post-Newtonian terms. An important advantage of the new method is that, contrary to the older KS-CHAIN code, zero masses are allowed (the AR-CHAIN code is either comparable in performance or superior to the regularization schemes based on the Kustaanheimo–Stiefel (KS) transformation, see [5] for the KS scheme). Moreover, it is also more efficient and shortens the computational times compared to previous algorithms (for the kind of problem we are trying to solve), such as the N-body ones we are using right now [3]. Now, we are also making some tests with AR-CHAIN. The next step should be restringing on PN1 terms and incorporate the cross ones, which, obviously, are not included in AR-CHAIN. This would be another way of exploration. One difficulty, whatsoever, is the implementation of star-star cross sum terms, as it requires be careful in the field computation on each body, which in some parts makes a triangular sum to accelerate the calculation. We are trying to design an strategy to incorporate them.

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3 Results

The inclusion of cross terms may significantly influence the system's evolution, suggesting a potential need to re-evaluate previous computational models. Until now, these models have not incorporated cross terms, which highlights a critical area for further investigation. The preliminary results obtained from our simulations, which now include these terms, appear promising and suggest substantial modifications to our existing understandings. In this study, we present these initial findings and explore their potential implications for advancing our knowledge of extreme mass ratio inspirals (EMRIs).

We would now comment some of the preliminary results we are obtaining, although in July (in the conference we should give in the Mathematical Modelling in Engineering & Human Behaviour (MME&HB2024) Meeting), we should have more research done and then present an updated collection of data and results.

Let us first analyse the meaning and importance of incorporating "cross terms". Such terms should be relevant for the case of extreme mass-ratio inspirals (EMRIs) and tidal disruption events. The study of the evolution of hierarchical triple systems suggests a deep exam of Post-Newtonian contributions into N-body treatments. So far, it is common to use relativity by adding to the Newtonian N-body equations the standard two-body post-Newtonian terms for a given star around the black hole or for the close binary in a triple system. According to Clifford Will [2] when timescales are of the order of the relativistic pericentre advance, the "cross terms" should be added in the equations of motion. Such terms represent a coupling between the potential of the Black Hole situated in the centre (massive or supermassive Black Hole) and other stars' potential considered in the system. The effects of these contributions may be "boosted" to amplitudes of the order of Newtonian ones. Clifford adds such Post-Newtonian contributions in the equations of motion in a truncated form which includes the cross terms that may contribute to the effects in an important way. Moreover, he gives the formulation for the direct implementation in the numerical algorithms. Clifford also states that for the conservation of total Newtonian energy Post-Newtonian cross terms are necessary. The particular simple case he studies to infer this need is the seemingly trivial case of the motion of a test body about a central body with a Newtonian quadrupole moment, who needs the inclusion of cross terms between the mass monopole potential and the quadrupole potential.

We will now expose some features of the equations of motion that seem to be important for the results we are obtaining in the consideration of the contribution of the cross terms. To do this, we would refer to equations (3.3) to (3.6) in the paper of Clifford Will (see pages 8 and 9 of [2]).

Let us start with the contribution of cross terms to the motion of the stars around the central Massive Black Hole. For this purpose, we begin with equation (3.3), the Post-Newtonian terms of order one (with the 1/c^2 factor, with c the light speed), PN1, are split into sums. In the following expressions, one has the usual PN1 field contribution, in equation (3.4a). Then, the PN1 cross term appears in equation (3.4b). An exhaustive look at all the sums that appear in equation (3.4b), which stands for the equation of motion of star "a", can tell us which kind of contributions one can expect. The following terms can be seen:

(i) The central Massive Black Hole (subindex 1) interaction on star "a" (subindexes "1a").

(ii) The central Massive Black Hole (subindex 1) interaction on star "b" (which is not the star interaction considered, but that also contributes to equation of motion of star "a", those are the "1b" terms).

(iii) The interactions between star “a” and the other stars, “b”, which are not the central Massive Black Hole (subindex 1), i.e., “ab” terms.

(iv) Notice that different sums over stars “b “ are performed.

(v) There are also contributions of velocities terms of the star “a”, but also of star “b” and even differences of velocities between star “a” and each of the other ones (subindexes “b”). In the preceding items the terms described are positions and not velocities.

(vi) In the truncated form presented by Cliflord Will, no PN1 star-star interaction different from the “a” and each of the other stars, “b”, are considered, as he exposes in the presentation of the equation.

We have made different computations including equation (3.4b) and the results seem promising as the interactions described seem affect the evolution of each star “a”. We compare the results obtained when considering only equation (3.4a) and those when taking the sum of equations (3.4a) and (3.4b). We found differences between both computations. Now, we are trying to understand those differences and relate them with the different terms that appear in equation (3.4b). Notice that if the differences are definitely confirmed they will alter the previous results obtained so far, as they only had in the formulae equation (3.4a). At present, we are checking the way the code works and if it is properly running. Moreover, different results are being obtained and also tested.

Now we will describe the new equations of motion for the central Massive Black Hole, which include the cross terms, and its implications. As it can be seen in equation (3.5) of [2] (similar to the case of star “a”), the central Massive Black Hole (subindex 1) Post-Newtonian contribution to first order, also PN1 terms, is split into two terms, the usual one, equation (3.6a), and the “new” cross term one, equation (3.6b).

As in the case of the bodies (each body “a”) orbiting the central one (body 1), we now study the sums that contribute to the central Massive Black Hole movement. Now, different for the standard treatment, there is a PN1 term (no cross term) which is not usually included in previous works. We are referring to equation (3.6a). We have launched runs including this term and it seems to contribute to the results obtained. In this way, the movement of the central Massive Black Hole is slightly different from the one computed so far and this also seems to have implications for the inferences made till present. We are making an exhaustive study of such implications. Notice that there is also a contribution of the velocities of all the stars (subindexes “b”) to the central body movement (body 1).

At this stage, we have to look at equation (3.6b) and describe the meaning of the different sums that appear in cross terms and that contribute to the movement of the central Massive Black Hole (body 1):

(I) The sum of each star (here is used the subindex “b” for all the bodies different from the central one) interaction on the central body (body 1), the “1b” subindexes.

(II) The inclusion of terms which have sums over two different stars (subindexes “bc”) and which are not the central one. This suggests that in the case of the central Massive Black Holes the star-star contribution (not that of the star-central star) should be considered. Those sums do not appear in the case of stars different from the central star (body 1), the equation (3.4b). This is a difficulty for computational programming, as it implies an “extra” sum over another one (an inner sum in an outer sum). This implies extra computational time waste, which we will try to reduce.

(III) There are also contributions of the velocities of bodies “b” and “c” and sums over those two stars (not the body 1, of course), but now, different from the other stars equation of motions, there are not relative velocities terms, “bc” velocities terms.

As explained in (II), the sums described there (“b”, “c”, stars, not central body) complicate the numerical implementation and may take additional computational time. Although till now we have only made an exhaustive study of a 3-body system. This fact has been taken into account in our numerical programming. Our first tests using more than 3 stars suggest that algorithms which accelerate computations (as the AR-CHAIN code, see [4]) will be more useful than more simple algorithms (such as those described in [3]).

For all the cases, as said before in star “a” study, we are checking that the code is working properly, now for the computation of the equation of motion of the central Massive Black Hole (the subindex 1 in Clifford formulae, paper [2]).

4 Conclusions

The results and implications about the knowledge and detection of EMRIs should be modify because of the consideration of cross terms (see reference [2]) in the calculation of PN1 contributions in the evolution of orbital systems around a Massive Black Hole. Our research team is now working in this field, using different N-body algorithms. In this work, we present our preliminary study and first results. As we will argue, more work on this line should be done. We present perspectives on this first research and propose these new lines of work. We will present new results in a near future.

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