Solving the puzzle game ICOXTREM

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1 Introduction

In this study, we present a collaborative endeavor between industry and academia. Specifically, it consisted in finding all the solutions of a puzzle-type game known as ICOXTREM.

In June 2022, the CEO of ICOXTREM S.L.U. reached out to the Department of Mathematics at the University of Castilla-La Mancha with a challenging problem. Several years prior, he had created ICOXTREM, a game comprising 20 pyramids with triangular bases joined along certain edges, resembling a snake of pyramids. The objective of the game is to assemble a convex regular icosahedron.

Before launching the product, he required certainty knowing the exact number of distinct solutions for the game, i.e., the various ways to construct an icosahedron. Despite discovering numerous solutions over the years through trial and error, the vast array of potential relative positions for the puzzle's 20 pieces left him uncertain if all possible solutions had been found, which meant an important setback. In fact, the game allows different solutions of different difficulty, and he needed to know all the solutions in order to devise a method which allows to differentiate them and, additionally, to order them according to their difficulty.

This collaboration led to the successful resolution of the posed problem through a mathematical and computational study of the game.

2 What is ICOXTREM?

ICOXTREM is a puzzle game comprising 20 pyramids with triangular bases (see Figure 1).

Each pair of consecutive pyramids is connected along an edge of their base triangles, highlighted in red in Figure 1, allowing for four distinct positions between them. With a total of 19 joint edges,

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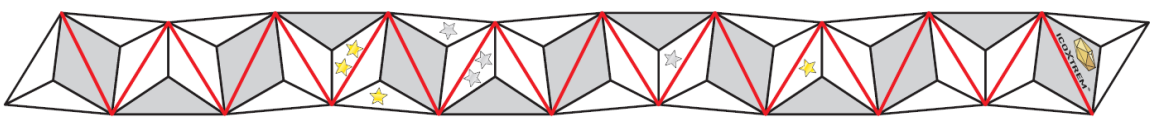

Figure 1: ICOXTREM game

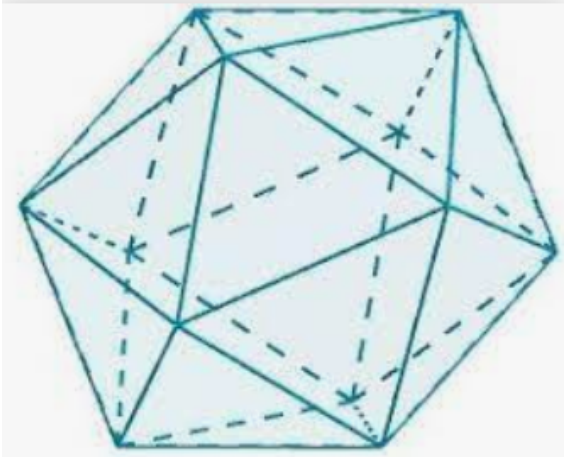

Figure 2: Icosahedron

there are theoretically 4^{19} different configurations of the puzzle. However, not all configurations are feasible, as the relative positioning of two consecutive pyramids may obstruct subsequent positions.

The objective of ICOXTREM is to assemble a convex regular icosahedron (see Figure 2).

Remember that the convex regular icosahedron belongs to the set of five regular Platonic solids, which also includes the tetrahedron, the cube, the octahedron, and the dodecahedron. The convex regular icosahedron is characterized by its 20 equilateral triangular faces, with five faces converging at each vertex.

3 Methodology and results

Due to the computational intractability of an exhaustive search given the vast number of potential configurations, we have developed a specialized search method to identify all solutions. This approach leverages geometric domain knowledge to effectively prune the search space while ensuring thorough exploration of feasible solutions. Our method combines a rule-based system with the Prolog backtracking search engine, taking into account the following key considerations:

- We establish a coordinate system aligned with the geometry of the icosahedron, encompassing all its symmetries.
- The potential positions of each pyramid within the icosahedron's geometry are determined relative to the placement of previous pyramids, ensuring that no pyramid occupies an already-occupied space.
- By exploiting the geometric sequence of pyramids, we can approach the problem by considering segments with identical structures, thereby significantly reducing the number of configurations to be examined.

By integrating these elements we successfully identify and describe all solutions of ICOXTREM, proving that there are exactly nine different solutions.

4 Conclusions

Solving a puzzle game using computational methods involves breaking down it into pieces to describe its solution(s) according to its rules and particularities [1]. It is an exercise in problem solving,

which can be extremely complex, where several variables may be tackled and different methodologies used to reach the solutions [2]. In this paper, we report the fruitful collaboration between the private company ICOXTREM S.L.U. and the University of Castilla-La Mancha to study the number and complete description of the solutions of a puzzle type game called ICOXTREM

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