# Approximations for Kemeny's Constant for several families of Graphs and Real-World Networks

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## Abstract

The linear relation between Kemeny's constant, a graph metric directly linked with random walks, and the effective graph resistance in a regular graph has been an incentive to calculate Kemeny's constant for various networks. In this paper we consider complete bipartite graphs, (generalized) windmill graphs and tree networks with large diameter and give exact expressions of Kemeny's constant. For nonregular graphs we propose two approximations for Kemeny's constant by adding to the effective graph resistance term a linear term related to the degree heterogeneity in the graph. These approximations are exact for complete bipartite graphs, but show some discrepancies for generalized windmill and tree graphs. However, we show that a recently obtained upper-bound for Kemeny's constant in [1] based on the pseudo inverse Laplacian gives the exact value of Kemeny's constant for generalized windmill graphs. Next, we have evaluated Kemeny's constant, its two approximations and its upper bound, for 243 moderate sized real-world networks. This evaluation reveals that the upper bound is tight, with average relative error of only 0.73%. In most cases the upper bound clearly outperforms the other two approximations [2].

For the upper bound based on the pseudo inverse of the Laplacian, we show that for a certain class of bimodal networks with diameter two, the upper bound is also tight. This generalizes previous results for bipartite and (generalized) windmill graphs. Moreover, we show numerically that also for large real-world networks this bound can be used to find good numerical approximations for Kemeny's constant. For certain graphs consisting of up to 100K nodes, we find a speed up of a factor 30 can be achieved, depending on the accuracy of the approximation. For networks consisting of over 500K nodes the approximation can be used to estimate values for Kemeny's constant, where exact calculation is no longer feasible within reasonable computation time.

## References

[1] Xiangrong Wang, J.L.A. Dubbeldam and P. Van Mieghem, Kemeny's constant and the effective graph resistance, *Linear Algebra and its Applications* 535:231-244, (2017)

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