A unified mathematical model for the late accelerated expansion and the early Universe's inflation

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Abstract

The standard ΛCDM model of cosmology has proven to be highly successful, encompassing the formation of elements, fluctuations of cosmic microwave radiation, the evolution of galaxies, and the large-scale structure of galaxies and clusters [11, 20]. However, despite its explanatory achievements, this model is still plagued by drawbacks, primarily due to our lack of understanding regarding the nature of its main components: dark matter and dark energy.

Conversely, the auxiliary assumptions of dark matter and dark energy are essential for achieving consistency with observations of both the early and late universe. Aligning the ΛCDM model with the inferred angular diameter distance to the surface of last-scattering necessitates a fraction $\Omega_\Lambda = 0.6911 \pm 0.0062$, as per the most recent findings from the Planck collaboration [16]. Furthermore, calibrating the cosmic distance ladder through Supernova Ia explosions also supports a flat universe where dark energy accounts for approximately 70% of the critical density [9].

These diverse observations collectively provide compelling evidence for the existence of dark energy. The prevailing notion is that dark energy originates from quantum fluctuations of the vacuum [19]. However, as noted previously, summing all normal modes of a field of mass m up to a large cutoff $M \gg m$ yields the energy density (in natural units $\hbar = c = 1$):

$$\langle \rho \rangle = \int_0^M \frac{4\pi k^2 dk}{(2\pi)^3} \frac{1}{2} \sqrt{k^2 + m^2} \simeq \frac{M^4}{16\pi^2}. \tag{1.1}$$

And, assuming that the sum must be carried out up to the Planck scale ($M = 1/\sqrt{8\pi G}$)

$$\langle \rho \rangle \approx 2^{-10} \pi^{-4} G^{-2} = 2 \times 10^{71}\,\mathrm{GeV}^4 \tag{1.2}$$

And this result is 118 orders of magnitude above the observed dark energy density,

$$|\rho_V| \lesssim 10^{-29}\ \mathrm{g/cm}^3 \approx 10^{-47}\,\mathrm{GeV}^4 \tag{1.3}$$

The problem with the preceding argument is that it presupposes that all particles are equally likely to appear in the quantum vacuum. This leads to this particular example of "ultraviolet catastrophe" in which the cosmological constant term is predicted as very large in comparison with the observations in standard cosmology.

To break the Gordian knot of this riddle, we will postulate that only fluctuations of the cosmic electromagnetic fields are relevant to the origin of dark energy. This makes the production of particles of mass m be suppressed by a Boltzmann factor of the form $e^{-mc^2/k_B T}$ [8]. Consequently,

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after recombination, which occurs at a Cosmic Microwave Background (CMB) temperature of $T = 3000$ °K [11], only the lightest bosons could be important to calculate the energy density of dark energy.

The lightest boson that could, possibly, exist in nature is the so-called axion that was proposed more than forty years ago to solve a fundamental problem in Quantum Chromodynamics (QCD) [12]. The motivation for the axion comes from the structure of the vacuum in QCD: Because the strong coupling constant is large at low energies, topological transitions are not suppressed. This gives rise to a superposition of states in the different θ-vacua, each of them characterized by a particular winding number [4, 11].

As a consequence, the QCD Lagrangian contains a term that violates CP symmetry (known as the θ term):

$$\mathcal{L}_\theta = \frac{\theta g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu} , \tag{1.4}$$

where g_s is the QCD gauge coupling, $G^a_{\mu\nu}$ is the gluonic field strength, and θ is a phase angle whose value is not predicted by the Standard Model [3]. On the other hand, the very small value of the neutron's electric dipole moment implies that $\bar{\theta} \equiv \theta - \arg\left(\det m_q\right) < 10^{-10}$, where m_q is the light quark mass [5]. This is the so-called "strong CP problem" of particle physics. Peccei and Quinn, in their theory, suggested that θ is a dynamical field that evolves to its lower value, very close to zero, to minimize its free energy [13, 14]. The quanta associated with this field are called axions following a proposal by F. Wilczek [21].

The axion mass has both an upper and lower bound. The lower bound, $m > 0.6 \times 10^{-5}$ eV, is determined by the fact that the energy density from axions cannot exceed the critical energy density. The upper bound, $m < 10^{-3}$ eV, is obtained from evidence such as the results from the supernova SN1987a and limits on the conversion of photons into axions via the Primakoff process in the core of stars [10]. These bounds help narrow down the possible range of the mass of the QCD axion. Despite ongoing experimental searches, there is currently no convincing evidence for the detection of axions. However, some authors have suggested that axions may have already been detected. For example, Beck proposed that the axion-induced electrical current in a Josephson junction could result in an additional supercurrent, leading to a Shapiro step anomaly when the axion field frequency aligns with the Josephson frequency corresponding to the applied voltage. This observation allowed Beck to estimate a mass of approximately 0.11 meV for the axion and an energy density near the Earth of $\rho_a = 0.051$ GeV/m^3 [1]. Another recent study proposed that the hard X-ray excess observed in certain nearby neutron stars, known as the " "magnificent seven", could be explained by the conversion of axions produced in the core into photons due to the strong magnetic fields surrounding the stars [2]. These authors found a mass of 0.02 meV for the axions emitted by the stars, which is an order of magnitude lower than Beck's result.

In this work we claim that axions, apart from being the main ingredient of dark matter, could also provide an explanation of dark energy in the late Universe after recombination. In this era, axions would be generated as virtual particles from fluctuations of the CMB. Then, we propose the following lagrangian density for the scalar field, $\hat{\varphi}$, associated with dark energy:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \hat{\varphi} \partial^\mu \hat{\varphi} + V(\hat{\varphi}) \ . \tag{1.5}$$

The potential function, $V(\hat{\varphi})$, is given by:

$$V(\hat{\varphi}) = \frac{1}{2} \frac{\hat{\varphi}^2}{\lambda^2} \mu^2(a) \ . \tag{1.6}$$

Here $\lambda = \hbar/(mc)$ is the Compton wavelength of the axions of mass m and $\mu(a)$ is a function of the cosmological radius. In our model the function $\mu(a)$ is inspired in the Planck's distribution:

$$\mu = \left(\frac{a}{a_0}\right)^{3/2} \left(\frac{mc^2}{k_B T}\right)^{3/2} \left[e^{mc^2/k_B T} - 1\right]^{-1/2} \ . \tag{1.7}$$

We have also that the equation of motion for this scalar field, $\hat{\varphi}$, is given by:

$$\ddot{\hat{\varphi}} + \frac{\hat{\varphi}}{\lambda^2} \mu^2(a) = -3 \frac{\dot{a}}{a} \dot{\hat{\varphi}} \ . \tag{1.8}$$

After solving Eq. (1.8), approximately, for φ we can arrive at the standard Friedmann equations for the scale factor, a, of the whole Universe. This is achieved by noticing that the stress-energy tensor of a scalar field is equivalent to that of an ideal fluid with density, ρ, and pressure, p, i.e.

$$\begin{aligned} \rho &= mc^2\left(\lambda^2\dot{\phi}^2 + \phi^2\right), \\ p &= mc^2\left(\lambda^2\dot{\phi}^2 - \phi^2\right). \end{aligned} \tag{1.9}$$

Here, as before, m is the mass of the boson and λ is its Compton wavelength. Finally, we find Friedmann equations for the cosmological radius:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_\Lambda + \text{Dark Matter term} + \text{Baryonic term}. \tag{1.10}$$

The density of dark energy in Eq. (1.10) is:

$$\rho_\Lambda = \frac{c^3\, m^4}{\hbar^3}\left(\frac{mc^2}{k_B T}\right)^3 \left(e^{mc^2/k_B T} - 1\right)^{-1} \psi_0^2\,. \tag{1.11}$$

Consequently, in our model, the density of dark energy depends upon the temperature, T, of the cosmic background. Here, m is the mass of the bosonic field that fluctuates in the quantum vacuum and ψ_0 is the normalized amplitude of that quantum field. In a static universe with constant temperature, T, we could also say that the density of dark energy is also constant. But, this is not the case as the temperature of our model will also decrease as the Universe expands.

The maximum energy density is achieved at the temperature:

$$k_B\, T_c \simeq 2.82144\, m\, c^2\,. \tag{1.12}$$

Therefore, an, approximately, exponential expansion is expected for $T \leq T_c$. If the corresponding virtual particle of the fluctuating quantum vacuum is the Higgs boson such accelerated expansion would take place in the very early Universe. Later on, we will have a slower accelerated expansion when the dominant contribution to the quantum vacuum comes from axions.

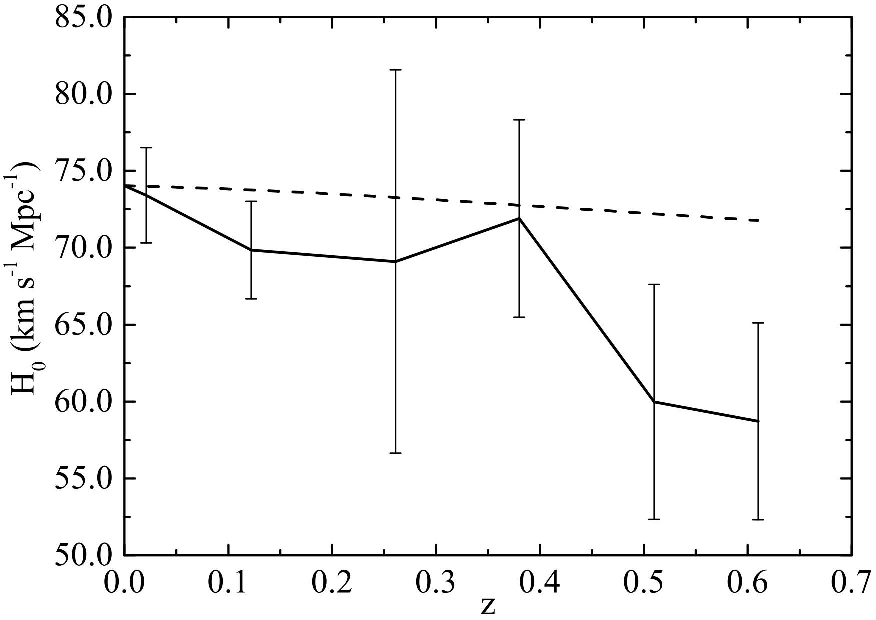

Figure 1.1: Fitting of the Hubble parameter to the ΛCDM model as a function of redshift obtained by Krishnan et al. [6] (solid line and error bars) compared with the predictions of the model in this work (dashed line).

Concerning the polemic of the Hubble parameter tension [15]: Our hypothesis is that the accurate value of the Hubble parameter corresponds to the one derived from the CMB, and the extrapolation from late-time measurements aligns with this data if $m \simeq 2.738\, k_B T_0 = 0.64$ meV. It is worth noting that this value falls within the permissible range for the QCD axion and is similar to the value discovered by Beck in his examination of Shapiro-step-like signals in Josephson junctions resonating with the axion mass [1]. Consequently, our solution corresponds to a dark energy that varies with time. Our model not only has implications for the early universe but also throughout its entire history. Wong et al. [22], from the H0LiCOW collaboration, have discovered some indications regarding the evolution of the Hubble constant's value as determined from the present time up to a specific redshift in the past. Krishnan et al. [6] analyzed cosmological data at low redshifts for $z < 0.7$ to provide a series of H_0 values obtained from fitting the ΛCDM model within the corresponding time interval. Their findings demonstrate a decreasing trend for H_0, which provides us with insights into the origin of the Hubble tension as a time-dependent phenomenon.

The H_0 trend discovered by the H0LiCOW project can also be fitted by our dark energy mathematical model as shown in Fig. 1.1.

On the other hand, it is already known that axions constitute a very promising dark matter candidate [17]. However, in contrast with the behaviour of massive weakly interacting particles or WIMPs, axions would characterize by forming Bose-Einstein condensates in galactic scales [18, 7]. Moreover, in this work, we have shown that virtual axions could also explain dark energy after the recombination era. If our model proves a viable theory, we could discover that the two most important riddles of modern cosmology are connected.

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